GRAVITY CAN'T EXIST IN 2 SPACETIME DIMENSIONS

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Post date: 25 Jan 2023.

One consequence of the Einstein equation is that gravity cannot exist in a vacuum in a universe with fewer than 4 dimensions (3 space and 1 time). Here we'll look at a demonstration of this for 2 dimensions of t and x.

Because of the symmetries of the Riemann tensor, there is only one independent, (possibly) non-zero component in 2 dimensions. We can take this component to be R_{xtxt}. We start with the Einstein equation

$$R^{ij}=\kappa\left(T^{ij}-\frac{1}{2}g^{ij}T\right)+\Lambda g^{ij} \tag{1}$$

For this argument, we'll assume $\Lambda=0$ (whether or not it is actually zero is still a subject of debate, but we do know it's very small). In a vacuum, the stress-energy tensor is zero (since there is no matter or energy), so in a vacuum $R^{ij}=0$. We'll now see what this implies about the Riemann tensor (which is the only thing we can use to show conclusively whether spacetime is flat or curved). If $R^{ij}=0$ then its lowered form is also zero:

$$R_{ij}=g_{ia}g_{jb}R^{ab}=0 \tag{2}$$

From the definition of the Ricci tensor

$$R_{ij} = R^{a}_{\ iaj} \tag{3}$$

$$= g^{ak}R_{kiaj} \tag{4}$$

$$= g^{tt}R_{titj}+g^{tx}R_{tixj}+g^{xt}R_{xitj}+g^{xx}R_{xixj} \tag{5}$$

Because $R_{ijkm}=-R_{jikm}=-R_{ijmk}$, any component with either the first two indices or last two indices equal is zero. Therefore

$$R_{tt}=0+0+0+g^{xx}R_{xtxt}=0 \tag{6}$$

$$R_{tx}=0+0+g^{xt}R_{xttx}+0=0 \tag{7}$$

$$R_{xt}=0+g^{tx}R_{txxt}+0+0=0 \tag{8}$$

$$R_{xx}=g^{tt}R_{txtx}+0+0+0=0 \tag{9}$$

All four of the Riemann components in these equations are either equal to R_{xtxt} or to $-R_{xtxt}$ so we see that this component must be zero (since all four components of the metric can't be zero). Therefore, the Riemann tensor is identically zero and space is flat in the vacuum. Flat space means no gravitational field, so gravity can't exist in two dimensions.

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