

HIGHER ORDER DERIVATIVES ARE NOT TENSORS

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 29 Jan 2021.

We've seen that the tangent to a curve is a contravariant tensor (vector, actually), since it transforms according to

$$\frac{dx'^{\alpha}}{du} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{dx^{\mu}}{du} \quad (1)$$

Also, the first derivative of a function is a covariant tensor, as it transforms according to

$$\frac{\partial g}{\partial x'^{\alpha}} = \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial g}{\partial x^{\mu}} \quad (2)$$

You might think that higher order derivatives are also tensors, but this turns out not to be the case. If we take the derivative of the last equation with respect to another of the primed coordinates x'^c , we get

$$\frac{\partial^2 g}{\partial x'^{\alpha} \partial x'^{\gamma}} = \frac{\partial^2 x^{\mu}}{\partial x'^{\alpha} \partial x'^{\gamma}} \frac{\partial g}{\partial x^{\mu}} + \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial^2 g}{\partial x^{\mu} \partial x'^{\gamma}} \quad (3)$$

$$= \frac{\partial^2 x^{\mu}}{\partial x'^{\alpha} \partial x'^{\gamma}} \frac{\partial g}{\partial x^{\mu}} + \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\nu}}{\partial x'^{\gamma}} \frac{\partial^2 g}{\partial x^{\mu} \partial x^{\nu}} \quad (4)$$

where we used the chain rule on the second term. In order for $\frac{\partial^2 g}{\partial x^{\mu} \partial x^{\nu}}$ to transform like a tensor, the first term in the last line would have to be zero, but it's clearly not, in general.

Higher order derivatives will also have extraneous terms, so it is only first derivatives that behave as tensors.

PINGBACKS

Pingback: [Christoffel symbols and the covariant derivative](#)

Pingback: [Electromagnetic field tensor - justification](#)