MACHOS AND SEEING DISTANT OBJECTS WITH A GRAVITATIONAL LENS

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Post date: 29 Sep 2022.

In gravitational lensing, the formula for the relative brightness of the images is given by

$$\frac{I_{\pm}}{I_s}=\frac{1}{4}\left(\frac{\sqrt{\beta^2+4\theta_E^2}}{\beta}+\frac{\beta}{\sqrt{\beta^2+4\theta_E^2}}\pm 2\right) \tag{1}$$

where $I_{\pm}$ are the brightnesses of the lensed images and I_s is the brightness of the unlensed source. We can write this as

$$\frac{I_{\pm}}{I_s}=\frac{1}{4}\left(\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{1/2}+\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{-1/2}\pm 2\right) \tag{2}$$

The combined brightness ratio of the two images is then

$$\frac{I_{tot}}{I_s}=\frac{I_+}{I_s}+\frac{I_-}{I_s} \tag{3}$$

$$=\frac{1}{2}\left(\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{1/2}+\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{-1/2}\right) \tag{4}$$

The two angles are given by

$$\theta_{\pm}=\frac{\beta}{2}\left(1\pm\sqrt{1+4\frac{\theta_E^2}{\beta^2}}\right) \tag{5}$$

so in cases where the two images are so close together they can't be distinguished, we see only the combined light intensity. The distance between the images is

$$\theta_+-\theta_-=\beta\sqrt{1+4\frac{\theta_E^2}{\beta^2}}=\sqrt{\beta^2+4\theta_E^2} \tag{6}$$

so if both β and θ_E are very small, the images will be indistinguishable. From the formula

$$\theta_E \equiv \sqrt{D_{LS}\frac{4GM}{D_L D_S}} \tag{7}$$

we see that θ_E will be small if S is much further away than L so that $D_{LS} \approx D_S$ but the distance to the lens still satisfies $D_L \gg 4GM$. The requirement that β is small means merely that S is not too far off being directly behind L.

Since θ_E is a constant for any given S and L, the total intensity depends only on β. From the formula above, we can show (by standard calculus) that $I_{tot}/I_s \geq 1$ for all values of β, so the lensing effect actually produces a brighter image than would be seen without the lens. The effect actually has a practical application in the detection of faint objects. If a lens moves transversely across the sky and passes nearly in front of a dim background source, the source will become brighter as the lens passes across it. Such lens objects are known as *massive compact halo objects* or MACHOs. To see how this works, suppose a MACHO moves at a constant angular speed so that its path across the sky as seen from earth is a straight line. We define β as the angle between the direction to the MACHO and the direction to the distant source, as usual. Let β_0 be the angle of closest approach, that is, the minimum of β as the MACHO passes near the source. Finally, we let α be the angle between the current position of the MACHO and its position when $\beta = \beta_0$. To a good approximation for small angles, the three angles β, β_0 and α form a right triangle, with β the hypotenuse, so

$$\beta^2 = \beta_0^2 + \alpha^2 \tag{8}$$

The angle α also represents the position along the path followed by the MACHO, so assuming a constant transverse motion, we have

$$\frac{d\alpha}{dt} = v_\alpha \tag{9}$$

for some constant v_α. If we let t_E be the time required for α to change by an angle θ_E, then

$$\frac{d\alpha}{dt} = \frac{\theta_E}{t_E} = v_\alpha \tag{10}$$

Therefore, if we define time $t = 0$ to be when $\beta = \beta_0$:

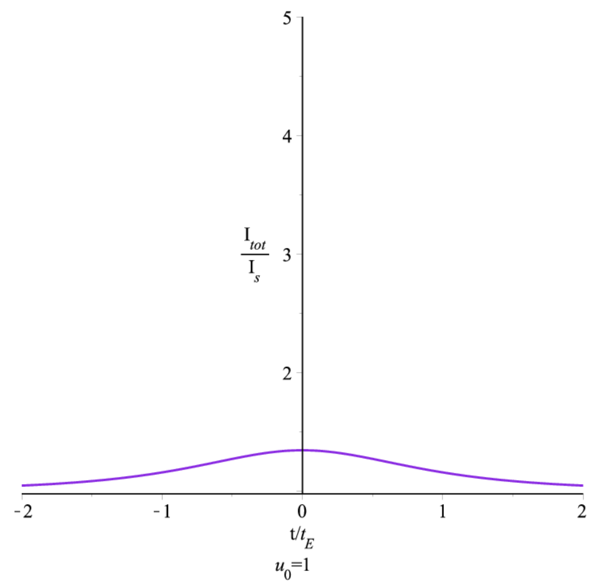

FIGURE 1. Plot of 15 for $u_0 = 1$.

$$\beta^2 = \beta_0^2 + \left(\frac{\theta_E t}{t_E}\right)^2 \tag{11}$$

$$= \theta_E^2 \left[\frac{\beta_0^2}{\theta_E^2} + \frac{t^2}{t_E^2}\right] \tag{12}$$

$$\equiv \theta_E^2 \left[u_0^2 + \frac{t^2}{t_E^2}\right] \tag{13}$$

with

$$u_0 \equiv \frac{\beta_0}{\theta_E} \tag{14}$$

Plugging this back into the equation above, we get

$$\frac{I_{tot}}{I_s} = \frac{1}{2}\left(q(t) + \frac{1}{q(t)}\right) \tag{15}$$

where

$$q(t) \equiv \sqrt{1 + \frac{4}{u_0^2 + t^2/t_E^2}} \tag{16}$$

Here are a few plots of I_{tot}/I_s are shown in Figs 1, 2 and 3.

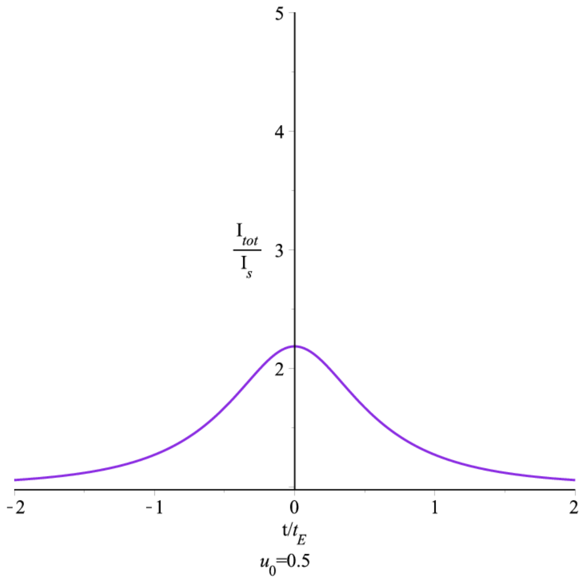

FIGURE 2. Plot of 15 for $u_0 = 0.5$.

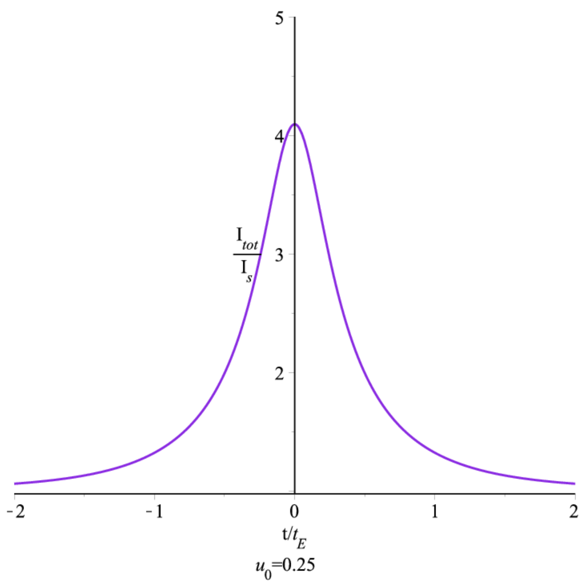

FIGURE 3. Plot of 15 for $u_0 = 0.25$.

We see that the smaller $u_0 = \beta_0/\theta_E$, the higher the peak brightness. For $u_0 = 0.25$, for example, the peak brightness is more than 4 times the unlensed image, so the effect is quite noticeable, and can be used to detect objects that would otherwise be invisible to earthbound telescopes.

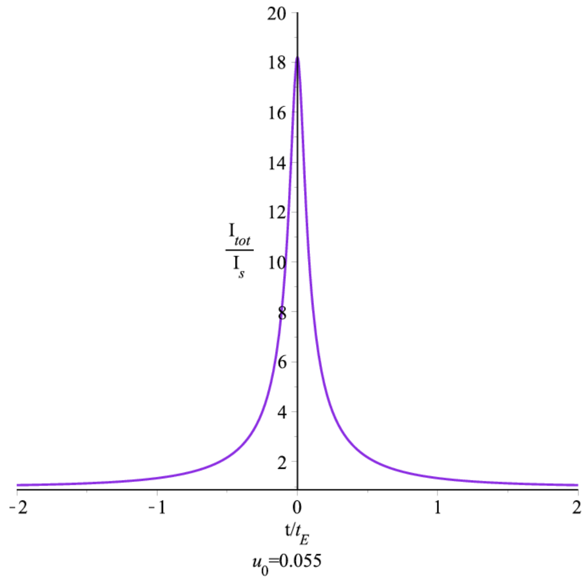

FIGURE 4. Plot of 15 for $u_0 = 0.055$.

A reasonable fit to Fig. 13.6 in Moore's book occurs when $u_0 = 0.055$ (Fig. 4).

To estimate t_E, we can observe the points at which the curve has the value $I_{tot}/I_s = 2$. In the plot from the formula, this happens at roughly $t/t_E = \pm 0.5$. In Fig 13.6, it happens at around $t = 568$ days and $t = 578$ days so $\Delta t = 10$ days and $\Delta t/t_E = 1$. Thus $t_E = 10$ days.

PINGBACKS

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