MEASURING THE SPEED OF LIGHT USING JUPITER'S MOONS

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One of the first (if not the first) determination of the speed of light was due to the Danish astronomer Ole Rømer in 1676. Since light moves much too fast for an earthbound measurement using the technology of the day, Rømer used an ingenious method based on timing the eclipses of Io, the innermost of the four Galilean moons of Jupiter.

The actual method used by Rømer is a bit involved, but we can get a good idea of how it works using a few simplifications.

Since Jupiter moves more slowly in its orbit than Earth, we can assume as a first approximation that Jupiter is stationary relative to Earth. Io's period P in its orbit about Jupiter is known to be about 42 hours. Because its orbital plane is very nearly the equatorial plane of Jupiter, Io gets eclipsed on each orbit, so it's possible to record the times at which Io becomes eclipsed with a fair degree of accuracy.

We'll define $t=0$ to be the time at which an eclipse of Io is observed, and t' to be the time at which the next eclipse is observed. If the linear speed of Earth in its orbit is v, then the distance Earth moves is about $v\Delta t=vt'$. This is an approximation since we're assuming that Earth is moving in a straight line over a 42-hour period, which should be reasonable given that Earth takes one year to complete one orbit.

If we take successive measurements of two eclipses to be at the point in Earth's orbit where it is heading directly towards Jupiter, then the observed time $t'=t_1$ of Io's period will be shorter than its actual value of P, because Earth moves a distance vt_1 closer to Jupiter between the two eclipses. By the same argument, if we take successive measurements when Earth is moving directly away from Jupiter, the measured time $t'=t_2$ will be longer than P. The light takes a time $\Delta t=\frac{vt'}{c}$ to travel the distance moved by Earth, so when Earth is moving towards Jupiter, the measured period P_1 is

$$t_1=P-\frac{vt_1}{c} \tag{1}$$

or

$$t_1 = \frac{P}{1+v/c} \tag{2}$$

When Earth is moving away from Jupiter, the measured period P_2 is

$$t_2 = P + \frac{vt_2}{c} \tag{3}$$

or

$$t_2 = \frac{P}{1-v/c} \tag{4}$$

Thus the measured periods differ by

$$\begin{aligned} \Delta t &= t_2 - t_1 & (5)\\ &= P\left(\frac{1}{1-v/c} - \frac{1}{1+v/c}\right) & (6) \end{aligned}$$

If we know v, Earth's speed in its orbit, and can measure Δt and P, we can get an estimate of the speed of light c.

To solve this, we'll define

$$\beta \equiv \frac{v}{c} \tag{7}$$

so we can write 6 as

$$\begin{aligned} \Delta t &= P\left(\frac{1}{1-\beta} - \frac{1}{1+\beta}\right) & (8)\\ &= P\frac{1+\beta-1+\beta}{1-\beta^2} & (9)\\ &= \frac{2\beta P}{1-\beta^2} & (10) \end{aligned}$$

This is a quadratic equation in β, with solutions

$$\beta = \frac{-P \pm \sqrt{\Delta t^2 + P^2}}{\Delta t} \tag{11}$$

Since we must have $\beta > 0$, we must take the plus sign, and we have

$$\beta = \frac{-P + \sqrt{\Delta t^2 + P^2}}{\Delta t} \tag{12}$$

In practice, Rømer measured Δt to be about 30 seconds, and P to be about 42 hours $= 1.512 \times 10^5$ seconds. Using the value $v = 3 \times 10^4$ m s^{-1}

for the speed of Earth in its orbit (which can be obtained if we know the Earth-Sun distance) gives us

$$\beta = \frac{v}{c} = 9.9 \times 10^{-5} \tag{13}$$

or

$$c = \frac{v}{\beta} = \frac{3 \times 10^{4}}{9.9 \times 10^{-5}} = 3.03 \times 10^{8} \text{ m s}^{-1} \tag{14}$$

which is quite close to the actual value of 2.99×10^{8} m s^{-1}.

The value obtained by Rømer was around 2.27×10^{8} m s^{-1} which is still not too bad, considering the difficulties in making accurate measurements and working out the details of the relative orbits of Earth and Jupiter, which we've glossed over here. It must be remembered that, at the time, it wasn't even known for sure that light had a finite speed.