

METRIC TENSOR AS A STRESS-ENERGY TENSOR

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 24 Jan 2023.

A curious possibility for the stress-energy tensor is

$$T^{ij} = -\Lambda g^{ij} \quad (1)$$

where Λ is a positive constant and g^{ij} is any metric tensor. In a local inertial frame (LIF), $g^{ij} = \eta^{ij}$ and

$$T^{ij} = \Lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (2)$$

Note that this tensor satisfies the energy conservation equation $\nabla_i T^{ij} = 0$ as the covariant derivative $\nabla_i g^{ij}$ is always zero.

Comparing this with the form of T^{ij} for a perfect fluid in its LIF

$$T^{ij} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{bmatrix} \quad (3)$$

we see that the energy density is $\rho = \Lambda > 0$ and the pressure is $P = -\Lambda < 0$. Such a tensor cannot arise from a perfect fluid because for such a fluid, for example

$$T^{xx} = \int \int \int N(p) \frac{(p^x)^2}{p^t V} dp^x dp^y dp^z \quad (4)$$

The integrand is intrinsically non-negative because $N(p)$ is the number of particles in volume V with a magnitude of momentum p and must be non-negative. The component $p^t = \gamma m$ for massive particles or $p^t = E$ for photons, but in either case it too is positive. The numerator $(p^x)^2$, being a square, is also non-negative. Thus for a perfect fluid in its LIF, $T^{xx} \geq 0$.

Although this tensor cannot be that of a perfect fluid, it does play a role in general relativity.

PINGBACKS

Pingback: [Stress-energy tensor - negative pressure revisited](#)

Pingback: [Dominant energy condition](#)