

METRIC TENSOR IN SEMI-LOG COORDINATES

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As an example of a different coordinate system, we can define the semi-log system by introducing coordinates p and q defined as:

$$p = x \quad (1)$$

$$q = e^{by} \quad (2)$$

where b is a constant. If x and y have units of length, then b must have units of length^{-1} . Note that this means that p and q have different units, with p having the units of length and q being dimensionless.

The curves of constant p are just the same as the curves of constant x , that is, vertical lines. The curves of constant q are defined by $e^{by} = k$ or $y = \frac{\ln k}{b}$. These are horizontal lines, although the spacing for equal steps in k will translate into the variable spacing seen on semi-log plots.

If an object has an acceleration $\mathbf{a} = a^i$ in the rectangular system, then we can find its acceleration in the semi-log system in the usual way.

$$a^p = \frac{\partial p}{\partial x} a^x + \frac{\partial p}{\partial y} a^y \quad (3)$$

$$= a^x \quad (4)$$

$$a^q = \frac{\partial q}{\partial x} a^x + \frac{\partial q}{\partial y} a^y \quad (5)$$

$$= b e^{by} a^y \quad (6)$$

The units of a^p are still those of acceleration, but the units of a^q are $\text{length}^{-1} \cdot \text{acceleration}$.

We can work out the metric of the semi-log system from the rectangular metric by direct calculation:

$$g'_{ij} = g_{kl} \frac{\partial x^k}{\partial x'^i} \frac{\partial x^l}{\partial x'^j} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{(bq)^2} \end{bmatrix} \quad (7)$$

The length squared of \mathbf{a} is an invariant, since

$$a^2 = g'_{ij} a'^i a'^j \quad (8)$$

$$= (a^x)^2 + \left(\frac{1}{bq}\right)^2 (be^{by} a^y)^2 \quad (9)$$

$$= (a^x)^2 + \left(\frac{1}{be^{by}}\right)^2 (be^{by} a^y)^2 \quad (10)$$

$$= (a^x)^2 + (a^y)^2 \quad (11)$$

The basis vectors in the semi-log system have lengths obtainable from the the metric:

$$\mathbf{e}_p \cdot \mathbf{e}_p = g_{pp} = 1 \quad (12)$$

$$|\mathbf{e}_p| = 1 \quad (13)$$

$$\mathbf{e}_q \cdot \mathbf{e}_q = g_{qq} = \frac{1}{(bq)^2} \quad (14)$$

$$|\mathbf{e}_q| = \frac{1}{bq} \quad (15)$$

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