POSTULATES OF SPECIAL RELATIVITY

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Einstein's special theory of relativity deals with motion in *inertial reference frames* so we'd better begin with a definition of such a frame. Introductory courses in relativity often define an inertial frame as one which is not accelerating, but this tends to hide the problem of how you tell whether a frame is accelerating, so we'll adopt a more formal definition.

An inertial frame must satisfy three properties:

(1) The distance d between a given pair of points P_1 with coordinates (x_1,y_1,z_1) and P_2 with coordinates (x_2,y_2,z_2) is independent of time.

(2) If we imagine a clock sitting at every point in the space (which requires an infinite number of infinitely small clocks, but never mind), then all these clocks are synchronized and they all run at the same rate.

(3) The geometry of all space at any particular time is flat, or Euclidean. This means that the distance between any two points can be found using the standard Euclidean distance formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \tag{1}$$

In fact, an exactly inertial frame doesn't exist, since in order for space to be Euclidean, there must be no gravitational fields, which means there has to be no matter or energy in the universe. However, as with so many things in physics, the inertial approximation is good enough for all practical purposes in situations where the gravitational field is weak and where acceleration can be kept to a minimum. In practice, we can imagine an inertial frame as a standard cartesian coordinate system moving at some constant velocity. Relative to what, you ask. Well, that's precisely the point, and leads us to consider the two postulates of special relativity.

First, there is the principle of relativity, which actually dates back to Galileo. It states that there is no such thing as an *absolute* velocity, since if such a velocity existed, it would require some absolute reference frame. Even though the principle of relativity underlies all of Newtonian physics, it wasn't fully accepted until Einstein's theory was validated. The Michelson-Morley experiment was still attempting to find some all-pervading ether which was the background substance which was postulated to fill all of space, and which gave light something to 'wave' in, in the same way that water provides the substance in which water waves can exist. The null result of the Michelson-Morley experiment killed off the notion of an ether and paved the way for 20th century physics.

So the principle of relativity really states that there is no preferred velocity or direction in space. A velocity has meaning only when it is measured relative to something else, so you can't assign some absolute value to an object's velocity.

Well, almost...

The second postulate of relativity states that there is one exception to the idea of a particle having an absolute velocity, and that is the case of light (or any electromagnetic radiation). The speed of light relative to any observer in any inertial reference frame is always the same numerical constant (about 3×10^8 m/sec in the MKS unit system). Notice that there is still no preferred *direction* for light to travel; space is still isotropic in that sense. However, the *speed* of light is always the same, no matter where or when it is measured.

Newcomers to relativity usually find both these postulates somewhat difficult to accept. Since we live on the surface of the Earth, we naturally tend to regard the Earth's surface as the ultimate frame of reference for our daily activities, and we therefore tend to measure all our velocities relative to it. The speed limits on roads are all implicitly assumed to be relative to the Earth's surface, for example.

However, there are some situations where the notion of relative speeds is a bit more obvious in daily life. If you are on a train that is stopped in a station and there is another train stopped on an adjacent track, when you see the train next to you move, it is often difficult to tell whether it is the train you are on or the one you can see through the window which is moving. Of course, again, there is the assumption that one or the other train is moving *relative to the Earth*, but if you consider just the two trains, you can for a brief moment anyway get some idea of the notion that it is impossible to tell which train is moving, and from the point of view of explaining what you see, it doesn't really matter.

Another common experience of relative motion is on an aircraft. A plane's speed is usually given in two forms: *air speed* and *ground speed.* Air speed is the plane's speed relative to the air through which it is moving, and ground speed is the speed relative to the ground below the aircraft. If the plane is encountering a headwind, then the air speed is greater than the ground speed, which is why it takes a plane longer to reach its destination in such a case. Again, though, we can relate air speed to ground speed if we know the wind speed, which is the speed of the air relative to the ground, so the notion of some absolute reference frame rears its ugly head again.

It is thus very difficult for us to rid ourselves of the idea that there has to be some absolute reference frame against which all velocities can be measured. We measure our activities relative to the Earth's surface. The Earth's surface has a (rather high) speed in its rotation about the Earth's axis. The Earth itself has a speed around the Sun, the Sun moves relative to the centre of the Milky Way galaxy, and the galaxy has a motion relative to other galaxies in the local galactic group. Surely if we extend this chain of reasoning far enough, we'll find some absolute property of the universe which can be used as the ultimate reference frame. This might be so, I suppose, but relativity *assumes* it's not true, and the theory based on this assumption has yet to fail any experimental test, so we're better off just accepting it.

The second postulate is even harder to swallow for most people, since it is so counter-intuitive. Suppose you are riding along as a passenger in a car and you throw a rock out of the car's window (not recommended, but just suppose). To get the velocity (speed and direction) of the rock relative to the ground you add (using vectors) the velocity of the car relative to the ground to the velocity of the rock relative to the car. To make things simple, suppose you throw the rock directly forward, in the same direction as the car is moving (and neglect air resistance, as all elementary physics problems seem to do). Then the speed of the rock is just the simple sum of the car's speed and the speed (relative to the car) with which you throw the rock. If the car is going 50 m/sec and you throw the rock at 10 m/sec relative to the car, the rock's ground speed is 60 m/sec. Easy. You can't find any fault in this logic.

However, the second postulate of relativity says that if you are in a car and then turn on the car's headlights, the speed of light relative to the car is given by that constant above (which is given the symbol $c = 3 \times 10^8$ m/sec, where the 'c' stands for 'celeritas', Latin for speed) *and* the speed of the same light relative to the ground (or relative to anything else, no matter how fast it's moving) is exactly the same. It's not $c + 50$ m/sec, it's still exactly c. In other words, the vector addition of velocities doesn't work when you're talking about light. (Actually, the vector addition doesn't work even at lower speeds either, but if the speeds are a lot less than c the difference isn't noticeable.) This seems completely wrong, but again has been verified in countless experiments, so it seems we must accept it.

Why did Einstein make this seemingly bizarre assumption in the first place? Clearly it doesn't follow from anything you would observe in your daily life. Well, basically, the speed of light is a constant quantity that comes out of Maxwell's theory of electromagnetism. In that theory, Maxwell predicts the existence of electromagnetic waves (which range from radio waves through the visible spectrum to X-rays), and their speed comes out of the equations as a constant. Maxwell himself recognized this, but he thought that there must be some absolute reference frame (the ether, for example) in which the speed of light has the value c, and that if you measured the speed of light in some frame moving relative to this absolute frame, you would get a different answer. This gave rise to the Michelson-Morley experiment, which tried to measure the speed of light in the direction of the Earth's motion and perpendicular to it. If the speed of light *did* depend on the motion relative to the ether, a difference should be detectable, but it wasn't.

Einstein's brilliant insight was that the best way to satisfy Maxwell's equations was to assume that there is no ether, and that the speed of light is an absolute constant, no matter how fast you're moving relative to the source of the light.

These two postulates, together with the definition of an inertial reference frame, are all that is needed to derive the mind-bending predictions of special relativity. The theory is remarkable not only for its revolutionary outlook on nature, but for the relative simplicity of the mathematics involved. Indeed many of the predictions can be made with nothing more complicated than a square root. Of course, when Einstein decided to extend the theory to include gravity, he paid for the simplicity of special relativity with mathematics of mind-boggling complexity that is needed in the gravitational theory.

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