RELATIVISTIC UNITS

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One of the key ideas drummed into the beginning physics student is that of checking the units in any calculation. If you are working out the value of some quantity such as the speed of a car or the kinetic energy of a falling mass, if the units don't work out to the correct combination for the quantity you're calculating, you've done something wrong.

The standard unit systems provide units for mass, distance and time, and the most commonly used systems in physics courses are the MKS system (metre-kilogram-second), also known as the SI (for Systeme Internationale) system, and the CGS system (centimetre-gram-second). Most other units used in physics are a combination of the three basic units and can be derived from the formula defining that particular unit.

For example, the unit of energy can be derived from the formula for kinetic energy, which is $T=\frac{1}{2}mv^{2}$. The unit of velocity is (distance)/(time), so the unit of energy is $(\text{mass})(\text{distance})^{2}(\text{time})^{-2}$. In the MKS system, this comes out to $\text{kg}\cdot\text{m}^{2}\text{s}^{-2}$ and the MKS unit of energy is named the joule.

This system of units works well for most practical applications of physics in the everyday world, since the units were chosen to be of magnitudes similar to those sorts of things we deal with on a daily basis. For most things, the quantities we need can be expressed with units that are within a factor of a thousand (larger or smaller) of the basic unit. Car speed limits, for example are expressed in kilometres per hour. The amount of a drug in a tablet is expressed in milligrams, and so on.

In relativity, however, we are pretty well always dealing with quantities that are well outside our daily comfort zone. Most speeds are sizeable fractions of the speed of light, which itself is $c=3\times10^{8}\text{ m s}^{-1}$. As a result, physicists working in relativity adopt a different unit system. It takes some getting used to, but if you do a lot of work in relativity, it can save a lot of time (and writing).

Since c is a fundamental constant, we can define it as having the value 1 and scale everything else to match. In fact, not only can we give c the numerical value of 1, we can also define it to be a dimensionless quantity. Now in the MKS system of units, of course, c is a velocity so it has units of (distance)/(time). If we want to define c as dimensionless, we are in effect saying that distance and time must be measured in the same units. This may sound crazy, but the way around it is to measure time in metres, where 1 metre of time is the time it takes light to travel one metre. Using this system of units, we can see that

$$\begin{aligned} c &= \frac{\text{distance light travels in time } t}{t} && (1)\\ &= \frac{\text{distance in metres}}{\text{time in metres}} && (2)\\ &= \frac{\text{distance in metres}}{\text{distance in metres}} && (3)\\ &= 1 && (4) \end{aligned}$$

So, no matter what distance we're talking about, c is always 1.

The effect of this is that time disappears as one of the fundamental units, so all quantities are now measured in units that are combinations of mass and distance. To convert to this relativistic system, only one conversion is needed: one second becomes the distance light moves in one second.

$$1 \text{ sec} = 3\times 10^8\text{m} \tag{5}$$

To get used to this system, a few examples come in handy.

1. Converting energy.

$$\begin{aligned} 1 \text{ joule} &= 1 \text{ kg m}^2\text{s}^{-2} && (6)\\ &= 1 \text{ kg m}^2(3\times 10^8\text{m})^{-2} && (7)\\ &= 1.11\times 10^{-17}\text{kg} && (8) \end{aligned}$$

2. Power. The watt is the unit of power and has the units of energy/time, or joule/sec. We can use the result from example 1 to get

$$\begin{aligned} 1 \text{ watt} &= 1 \text{ joule s}^{-1} && (9)\\ &= 1.11\times 10^{-17}\text{kg}(3\times 10^8\text{m})^{-1} && (10)\\ &= 3.704\times 10^{-26}\text{kg m}^{-1} && (11) \end{aligned}$$

3. Planck's constant $\hbar = h/2\pi = 1.05\times 10^{-34}$joule s. This has units of (energy)(time) so we again use the result of example 1 to get

$$\begin{aligned} 1.05\times 10^{-34}\text{joule s} &= 1.05\times 10^{-34}(1.11\times 10^{-17}\text{kg})(3\times 10^8\text{m}) && (12)\\ &= 3.5\times 10^{-43}\text{kg m} && (13) \end{aligned}$$

4. Velocity of a car $v = 30\text{m s}^{-1}$.

$$30\text{m s}^{-1} = 30\text{m}(3\times 10^{8}\text{m})^{-1} \tag{14}$$
$$= 10^{-7} \tag{15}$$

Note that velocities are dimensionless. The numerical value should be interpreted as a fraction of the speed of light, so in this case, a car travelling at 30 m/sec is going at one ten-millionth the speed of light.

5. Momentum of a car: $3\times 10^{4}\text{kg m s}^{-1}$.

$$3\times 10^{4}\text{kg m s}^{-1} = 3\times 10^{4}\text{kg m}(3\times 10^{8}\text{m})^{-1} \tag{16}$$
$$= 10^{-4}\text{kg} \tag{17}$$

Note that momentum and energy both have the same units (mass).

6. Pressure of one atmosphere: $10^{5}\text{newton m}^{-2}$. The newton is the unit of force, which from Newton's law has the units of (mass)(acceleration). Acceleration has the units of $(\text{distance})/(\text{time})^{2}$, so we get

$$10^{5}\text{newton m}^{-2} = 10^{5}\text{kg m s}^{-2}\text{ m}^{-2} \tag{18}$$
$$= 10^{5}\text{kg m}^{-1}(3\times 10^{8}\text{m})^{-2} \tag{19}$$
$$= 1.11\times 10^{-12}\text{kg m}^{-3} \tag{20}$$

7. Density is measured in units of (mass)/(volume). Since this does not contain any units of time, density units remain the same in the relativistic system.

8. Luminosity flux: $10^{6}\text{joule s}^{-1}\text{cm}^{-2}$. This contains a mixture of MKS and CGS units, so we need to convert it to MKS first. Using the conversion $1\text{ cm}^{2} = 10^{-4}\text{m}^{2}$ we get:

$$10^{6}\text{joule s}^{-1}\text{cm}^{-2} = 10^{10}\text{joule s}^{-1}\text{m}^{-2} \tag{21}$$
$$= 10^{10}(1.11\times 10^{-17}\text{kg})(3\times 10^{8}\text{m})^{-1}\text{m}^{-2} \tag{22}$$
$$= 3.7\times 10^{-16}\text{kg m}^{-3} \tag{23}$$

Note that pressure, density and luminosity flux all have the same units.

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