RICCI TENSOR AND CURVATURE SCALAR

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We can form contractions over the indices of the Riemann tensor to get some other useful quantities.

First, we can contract the first and third indices to get the *Ricci tensor* R_{bc}:

$$R^a_{\ bac} = g^{ad} R_{dbac} \tag{1}$$

$$\equiv R_{bc} \tag{2}$$

Using the symmetry relation $R_{dbac} = R_{acdb}$ and the symmetry of the metric, we have

$$R_{bc} = g^{ad} R_{dbac} = g^{da} R_{acdb} = R_{cb} \tag{3}$$

so the Ricci tensor is symmetric.

We can contract the Ricci tensor in turn to get the *curvature scalar* R:

$$R^b_{\ c} = g^{ab} R_{ac} \tag{4}$$

$$R^b_{\ b} = g^{ab} R_{ab} \tag{5}$$

$$\equiv R \tag{6}$$

Since the Riemann tensor is identically zero in flat spacetime, the Ricci tensor and curvature scalar are also both zero there. However, although the Riemann tensor always has at least one non-zero component in curved spacetime, the Ricci tensor and curvature scalar can both be zero in curved spacetime. Thus we can say that if the Ricci tensor or the curvature tensor are non-zero, the spacetime is curved, but we can't draw any conclusions if they are zero; we then need to work out the full Riemann tensor.

One other contraction of the Riemann tensor is over its first and second indices:

$$R^a_{\ abc} = g^{ad} R_{dabc} = -g^{da} R_{adbc} = 0 \tag{7}$$

In the second equation we've used the antisymmetry relation $R_{dabc} = -R_{adbc}$ and the symmetry of the metric. Thus this contraction doesn't tell us anything useful.

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