SPACETIME DIAGRAMS - PARTICLE DETECTOR

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We look at an experiment in which one particle is emitted at $(t,x) = (-2,0)$ in the unprimed $\mathcal{O}$ frame in the $-x$ direction at a speed of 0.5, and another particle is emitted at the same event, but travels with speed 0.5 in the $+x$ direction. Each particle is absorbed by a detector located at $x = \pm 2$. After a time interval of 0.5 (as measured by $\mathcal{O}$), the detectors send a signal travelling at a speed of 0.75 back to $x = 0$. The experiment is shown in a spacetime diagram in Fig. 1

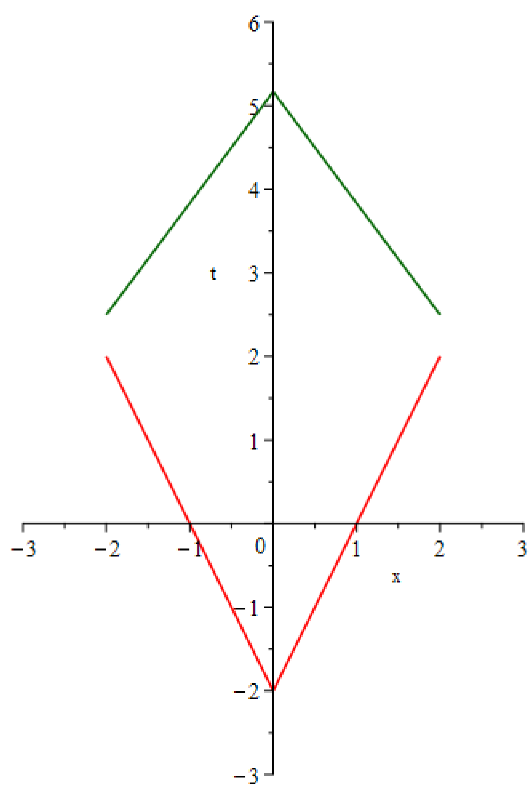

FIGURE 1. Particles emitted and detectors send signals.

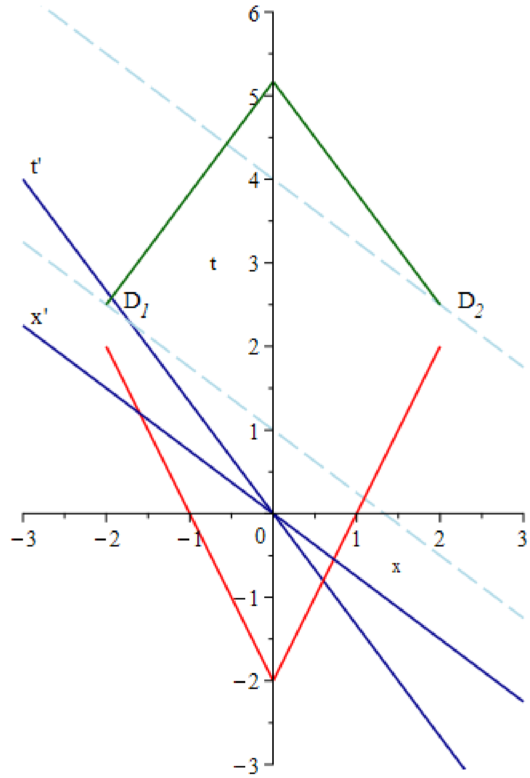

FIGURE 2. The experiment viewed by $\bar{\mathcal{O}}$.

The emitted particles' world lines are shown in red, and have slopes of $\pm\frac{1}{0.5} = \pm 2$. The signals sent by the detectors are shown in green and have slopes of $\pm\frac{1}{0.75} = \pm\frac{4}{3}$.

As can be seen from the diagram, the two signals arrive back at $x = 0$ at the same time. We can conclude that the detectors emitted their signals at the same time (as measured by $\mathcal{O}$), since they are equally distant from $x = 0$, and the detectors are at rest in $\mathcal{O}$'s frame.

Now consider an observer $\bar{\mathcal{O}}$ moving at velocity $v = -0.75$ (in the $-x$ direction) relative to $\mathcal{O}$. The x' and t' axes of $\bar{\mathcal{O}}$ therefore have slopes of -0.75 and $-\frac{1}{0.75}$ respectively, as shown in Fig. 2.

The x' and t' axes are shown in dark blue. The events at which the detectors emitted their signals are shown as points D_1 and D_2. To find the times at which these events occur in $\bar{\mathcal{O}}$'s frame, we need to draw lines through D_1 and D_2 parallel to the x' axis and find where they intersect the t' axis. These lines are shown as the light blue dashed lines. We can see that the D_1 line intersects the t' axis quite close to the D_1 event. The intersection between the D_2 line and the t' axis occurs out of the range of the graph, far to the upper left. Thus the time t_1' of D_1 as measured by $\bar{\mathcal{O}}$ is *before* time t_2', and $\bar{\mathcal{O}}$ says that detector D_1 emitted its signal first.

We can put in some numbers to verify that the spacetime interval between D_1 and D_2 is the same as measured by both observers. In $\mathcal{O}$'s frame, we have for the events D_1 and D_2:

$$\begin{aligned}(t_1,x_1) &= (2.5,-2)\\ (t_2,x_2) &= (2.5,2)\end{aligned} \tag{1}$$

We can use Lorentz transformations to find the corresponding values in $\bar{\mathcal{O}}$'s frame. We have

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-0.75^2}} = 1.512 \tag{2}$$

The transformations are then

$$\left(t_1',x_1'\right) = \left(\gamma\left(t_1-vx_1\right),\gamma\left(x_1-vt_1\right)\right) \tag{3}$$
$$= (1.512,-0.189) \tag{4}$$
$$\left(t_2',x_2'\right) = \left(\gamma\left(t_2-vx_2\right),\gamma\left(x_2-vt_2\right)\right) \tag{5}$$
$$= (6.047,5.858) \tag{6}$$

The invariant interval as calculated by $\mathcal{O}$ is

$$\Delta s^2 = -\Delta t^2 + \Delta x^2 = 16 \tag{7}$$

From $\bar{\mathcal{O}}$'s frame, we have

$$\Delta s'^2 = -(6.047-1.512)^2 + (5.858+0.189)^2 = 16 \tag{8}$$

Thus the two intervals are the same.