# SPACETIME DIAGRAMS

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A central concept in special relativity is that of an *event*. An event is anything which can be given a specific time and location by a given observer. One event could be the front end of a car of length $l=2$ m is observed to be at location $x=0$ at time $t=0$. Another event, assuming that the car is moving to the right, is that the back end of the car is at location $x=0$ at time $t=10^7$ Remember that in relativistic units, both time and distance are measured in metres, with the time being the time that light takes to travel the given distance. Thus if a car is travelling 60 m/s, its relativistic speed is $v=60/3\times10^8=2\times10^{-7}$. At that speed, it would take a time of around $t=l/v=2/2\times10^{-7}=10^7$ m for the back end of the car to pass the point $x=0$. (We've ignored the relativistic length contraction effect here, but at a car's speed, that is negligible anyway.) In old units (that is, seconds), this time is the time it takes light to travel $10^7$ m, which is $10^7/3\times10^8=0.033$ seconds.

The key point is that an event has a specific time and location as measured by each observer. An event has no duration in time or extent in space; it occurs at precisely one location and one time. Thus it makes no sense to say that an event is 'the car passes my location' since a car is an extended object, so you need to specify which part of the car you're talking about.

When learning relativity for the first time (or even later), a *spacetime* diagram is a useful aid. To simplify things, we'll assume that motion can occur in only one dimension, which we'll call the $x$ direction. A spacetime diagram allows events to be plotted as points on the diagram, where each point's coordinates specify the values of position ($x$) and time ($t$) at which that event occurred. By convention, the $x$-axis is horizontal and the $t$-axis is vertical. A simple spacetime diagram is shown in Fig. 1.

Suppose we have a rigid rod of length 3 m. If we align this rod so that it lies along the $x$-axis with its left end at $x=0$ and its right end at $x=3$, and keep the rod at rest relative to us, then clearly the left end will always have the location $x=0$ and the right end will always be at $x=3$. Suppose we observe the location of the rod at time $t=0$. The left end of the rod will be at coordinates $(x,t)=(0,0)$ and the right end will be at $(x,t)=(3,0)$. Since these two points each have precise values of $x$ and $t$, they are both

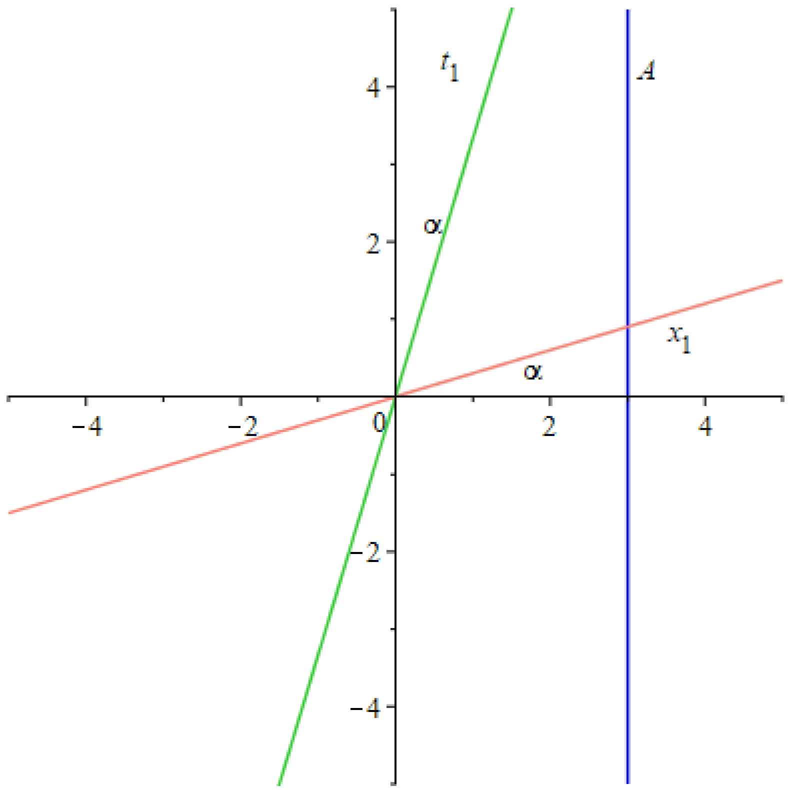


FIGURE 1. Spacetime diagram.

events. We can draw these events on the spacetime diagram, and they will appear at the origin and the location (3,0) respectively. Note that the latter point is where the vertical blue line crosses the $x$-axis.

Now suppose we wait until time $t = 1$ and observe the rod again. Since it's at rest in our frame of reference, its $x$ coordinates will not change, so its left end is now at (0,1) and its right end is now at (3,1). These are two more events. Notice that events, in general, aren't very exciting occasions; they merely define where and when something is. Clearly if we just sit there and make repeated observations of our rod that is at rest, its location on the $x$-axis will never change, but the time coordinate will advance at a steady rate. If we connect up the sequence of events that define where the left end of the rod is found, we will trace out the vertical ($t$) axis from the origin upwards. The other end of the rod will trace out the vertical blue line (labelled $A$ in the diagram) that passes through the point (3,0). If the rod was at rest before $t = 0$, then the continuation of these two lines below the $x$-axis describes the events corresponding to the endpoints of the rod before this time.

Now suppose we have another observer who is moving along the $x$-axis at a constant velocity $v$. The constant velocity assumption is needed if this observer is to be in an inertial reference frame. What would this observers path look like on our spacetime diagram?

To make things simple, suppose the observer (who we will assume is infinitesimal in size, so we can assign a single event to him being at a particular point at a particular time) passes our $x=0$ at our time of $t=0$. Since he is moving with speed $v$ to the right, then after a time $t'$, he will have moved a distance $vt'$, so the events corresponding to the location of the observer will all be of the form $(vt',t')$. This is the parametric form of a straight line:

$$t = t' \quad (1)$$

$$x = vt' \quad (2)$$

$$t = \frac{x}{v} \quad (3)$$

where in the last line, we just eliminated the parametric variable $t'$ to get the equation of the straight line. We can see that the line passes through the origin and has slope $1/v$. It is the green line labelled $t_1$ in the diagram. Note that as $v$ gets smaller, the slope gets larger and the line $t_1$ gradually gets closer to the vertical $t$-axis. If $v=0$, the slope is infinite and we get back the vertical line which is the path of a stationary object in the spacetime diagram.

Also notice that the tangent of the angle $\alpha$ between the $t$-axis and the line $t_1$ is just $v$, the relative velocity of the two observers. To see this, drop a perpendicular from the line $t_1$ to the left so that it intersects the vertical axis, and use the definition of the tangent on the right-angled triangle thus formed. That is

$$\tan\alpha = v \quad (4)$$

You may have noticed that we drew in the second observer's $x_1$ axis at an angle of $\alpha$ to the original $x$ axis (red line in Fig. 1). We'll justify this in another post.

One final observation at this point. Since the speed of light is $c=1$, and since light always moves at this constant speed relative to any observer, then the path of a light ray in a spacetime diagram is always at an angle given by $\tan\alpha = \pm 1$ (the negative sign allows for light rays moving to the left, the positive sign to the right). That is, in a spacetime diagram, light rays *always* are drawn at an angle of 45°. The restriction of all relative speeds to be less than that of light means that the only physically realizable paths for an object are those lines between the 45-degree line and the vertical axis.

Thus a line such as that labelled $x_1$ in the diagram is not a possible path for any real object.

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