STRESS-ENERGY TENSOR FOR PERFECT FLUID - GENERAL COORDINATES

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We worked out the stress-energy tensor T^{ij} for the case of a perfect fluid in its rest frame, so now we want to generalize this to the case where the fluid is viewed from some more general coordinate system, possibly in curved spacetime. The result is simply stated in Moore's equation 20.16 and in pretty well every other source I looked at. The result is

$$T^{ij}=(\rho_0+P_0)\,u^iu^j+P_0g^{ij} \tag{1}$$

where ρ_0 is the energy density of the fluid and P_0 is the pressure, both measured in the fluid's rest frame. We can verify that this reduces to the form we derived earlier when viewed in the fluid's rest frame, since in that frame, $g^{ij}=\eta^{ij}=[-1,1,1,1]$ and the four-velocity is $u^i=[1,0,0,0]$, so all off-diagonal elements are automatically zero, and

$$T^{tt}\,(\text{rest frame})=(\rho_0+P_0)\,u^tu^t-P_0=\rho_0+P_0-P_0=\rho_0 \tag{2}$$

$$T^{xx}\,(\text{rest frame})=T^{yy}=T^{zz}=(\rho_0+P_0)\times 0+P_0\times 1=P_0 \tag{3}$$

$$T^{ij}=\left[\begin{array}{cccc}\rho_0 & 0 & 0 & 0\\ 0 & P_0 & 0 & 0\\ 0 & 0 & P_0 & 0\\ 0 & 0 & 0 & P_0\end{array}\right] \tag{4}$$

The justification of 1 appears to be that it is a proper tensor equation (since u^i is a four-vector and g^{ij} is a tensor, and ρ_0 and P are both scalars), so it is valid in all coordinate systems. The fact that it reduces to the earlier form in the fluid's rest frame shows that it has the right form in that coordinate system. What isn't clear is that this is the *only* tensor that reduces to the correct form. Anyone who knows of a proper derivation, please leave a comment.

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