

## TENSOR TRACE

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The *trace* of a rank-2 tensor is given by the contraction  $F^i_i$ . In matrix terminology, it is the sum of the diagonal elements. If we start with a contravariant tensor  $F^{ij}$ , then we can calculate the trace as follows:

$$F^i_j = g_{jk} F^{ik} \quad (1)$$

$$F^i_i = g_{ik} F^{ik} \quad (2)$$

That is, we first lower the second index, then contract the top and bottom indices.

Since the trace contains no free index, it should be a scalar, which means it should be invariant. We can prove this by doing the transformation.

$$g'_{ik} F'^{ik} = \frac{\partial x^l}{\partial x'^i} \frac{\partial x^m}{\partial x'^k} g_{lm} \frac{\partial x'^i}{\partial x^a} \frac{\partial x'^k}{\partial x^b} F^{ab} \quad (3)$$

$$= \left( \frac{\partial x^l}{\partial x'^i} \frac{\partial x'^i}{\partial x^a} \right) \left( \frac{\partial x^m}{\partial x'^k} \frac{\partial x'^k}{\partial x^b} \right) g_{lm} F^{ab} \quad (4)$$

$$= \delta^l_a \delta^m_b g_{lm} F^{ab} \quad (5)$$

$$= g_{ab} F^{ab} \quad (6)$$

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