TIME DILATION AND PROPER TIME

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One bizarre effect of the postulates of special relativity is *time dilation*. To see how this comes about, we do the usual thought experiment of two observers, with one travelling with a speed v relative to the other stationary observer. To help visualizing the situation, suppose that the moving observer A is on a train, and the height of the train's compartment is h. Observer B views the train from beside the track.

Now suppose there is a light bulb on the ceiling of the train's compartment. If the bulb is switched on, the light takes a time $t_A = h/c = h$ (since we're using relativistic units where the speed of light $c = 1$) to reach the floor of the train, as seen by A who is on board the train. The path of the light is perpendicular to the relative motion of A and B, and lengths perpendicular to the motion are not affected.

Observer B sees the light take a diagonal path to reach floor. The speed of light is the same for both observers, and the path of the light is the hypotenuse of a right-angled triangle with vertical distance h as seen by both observers. Suppose the time taken for the light to reach the floor as seen by B is t_B. In that time, the train will move vt_B relative to B, so that is the horizontal side of the triangle. The hypotenuse has length t_B (remember $c = 1$) so using Pythagoras:

$$t_B^2 = h^2 + (vt_B)^2 \tag{1}$$

which can be solved for t_B:

$$t_B = \frac{h}{\sqrt{1-v^2}} = \gamma h = \gamma t_A \tag{2}$$

That is, $t_B > t_A$ so B thinks the light takes longer to reach the floor than A does. This is the phenomenon of time dilation. The time t_A is the time as measured by a observer stationary with respect to the events being witnessed (in this case, A is stationary relative to the train on which the experiment takes place). This time is known as the *proper time*, and is usually given the symbol τ. Since any clock moving relative to A measures a longer time, we see that proper time is the minimum time that can be measured between two events. A popular way of expressing this is that 'moving clocks run slow'. That is, a clock moving relative to you will show less time elapsing than a clock (such as a wristwatch) attached to you.

Time dilation also follows from Lorentz transformations. The transformations for intervals are (since T is moving to the right with speed v relative to G):

$$\begin{aligned}\Delta t_B &= \gamma(\Delta t_A + v\Delta x_A)\\ \Delta x_B &= \gamma(\Delta x_A + v\Delta t_A)\end{aligned} \tag{3}$$

In our train example, observer A is stationary relative to the train, so $\Delta x_A = 0$ and we have

$$\Delta t_B = \gamma\Delta t_A \tag{4}$$

as before.

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