TEMPERATURE

Although we’re all familiar with temperature, it’s quite difficult to give a precise definition of it. To get started, we can look at the notion of thermal equilibrium. Two objects are in thermal equilibrium if, when they are in contact, there is no net energy transferred from one to the other. This allows us to define temperature in a relative sense: if there is a net spontaneous transfer of energy from object $A$ to object $B$, then the temperature $T_A$ of $A$ is higher than the temperature $T_B$ of $B$.

To attach units to temperature we can pick some common substance such as water and consider its freezing and boiling points at standard atmospheric pressure and assign some numbers to the temperature of water at these two points. The Celsius (centigrade) scale assigns 0 to the freezing point and 100 to the boiling point. We can then insert a thermometer, such as a mercury thermometer, into the water and mark the points 0 and 100 on the tube. We can then divide up the portion of the tube between these two points into 100 equally spaced intervals, thus defining the temperature of any other object into which we place the thermometer (extending the marks below 0 and above 100 as required). This technique has obvious limitations; mercury freezes at $-38.8^\circ C$ so the thermometer won’t be much use below that point. At the other extreme, the glass tube will become soft and deform at some point, and mercury boils at $356.7^\circ C$. We’ve also made the assumption that the expansion rate of mercury is constant over the range of the thermometer, and so on.

A thermometer can also be made using the expansion and contraction of a gas with temperature. Experimentally, gases obey the ideal gas law over a wide range of pressures and temperatures. The law says

$$PV = nRT$$

where $P$ is the pressure, $V$ the volume, $n$ is a measure of the number of gas molecules in the container, $T$ is the temperature (in kelvin) and $R$ is the gas constant. The kelvin scale is obtained from Celsius scale by adding 273.15 to the latter:
\[ K = C + 273.15 \]  

Thus absolute zero is 0 K = −273.15 C.

**Example 1.** The Fahrenheit scale defines the freezing point of water to be 32°F and the boiling point to be 212°F. (The origin of these rather bizarre values, or more to the point, the reason why 0°F is where it is, seems rather obscure; see the [Wikipedia article](https://en.wikipedia.org/wiki/Fahrenheit) if you’re interested.) This means that there are 180 Fahrenheit degrees between the freezing and boiling points, so there are \( \frac{180}{100} = \frac{9}{5} \) Fahrenheit degrees per Celsius degree. Thus to convert from F to C we first subtract 32, then multiply by \( \frac{5}{9} \):

\[ C = \frac{5}{9}(F - 32) \]  

or, the other way round:

\[ F = \frac{9}{5}C + 32 \]

Thus absolute zero is \( \frac{9}{5}(-273.15) + 32 = -459.67°F \).

An approximate formula that works fairly well for temperatures in the 0 to 30 C range is to double the C value then add 30 to get the F value (or, conversely, subtract 30 from F then divide by 2 to get C). The temperature of −40 is the same on both scales (so 40 below is darned cold no matter how you measure it!).

**Example 2.** The Rankine temperature scale uses Fahrenheit-sized degrees, but its zero is at absolute zero, so that

\[ R = F + 459.67 \]  

\[ = \frac{9}{5}C + 491.67 \]  

\[ = \frac{9}{5}K \]

Room temperature (21 C, say) is thus 529.47 R.

**Example 3.** Some examples of kelvin temperatures are

<table>
<thead>
<tr>
<th>Celsius</th>
<th>Kelvin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human body temp</td>
<td>37</td>
</tr>
<tr>
<td>Boiling point of water</td>
<td>100</td>
</tr>
<tr>
<td>A very cold day (in Dundee, anyway)</td>
<td>−10</td>
</tr>
<tr>
<td>Boiling point of nitrogen</td>
<td>−196</td>
</tr>
<tr>
<td>Melting point of lead</td>
<td>327</td>
</tr>
</tbody>
</table>
Example 4. It makes sense to say one object is “twice as hot” as another if we’re using the kelvin (or Rankine) scale, since the absolute zero of temperature is zero on these scales. Saying that 20°C is twice as hot as 10°C is wrong, of course. That’s like measuring a person’s height by defining the zero height to be at an ‘absolute’ height of 150 cm. Using that definition, we wouldn’t say that someone with an absolute height of 160 cm is twice as tall as someone with a height of 155 cm.

Example 5. The relaxation time is, roughly speaking, the time required for two objects initially at different temperatures to come to thermal equilibrium when placed in contact. Mathematically, the temperature difference declines as $\Delta T = \Delta T_0 e^{-At}$ for some constant $A$, so it’s more precise to define relaxation time as the time required for the temperature difference to decrease to specified fraction (say $1/e$) of its initial value $\Delta T_0$. When you take your temperature by putting a fever thermometer under your tongue, you typically have to wait around 2 minutes before taking a reading, so that’s the relaxation time between objects $A$ (your mouth) and $B$ (the thermometer).

Example 6. The human sense of touch is notoriously bad at being able to judge temperature. A common experiment involves placing one of your hands in a bowl of cold water and the other in a bowl of hot (not too hot!) water. Leave your hands there for a minute or two and then touch an object at room temperature with each hand in turn. The cold hand will sense the object as warm, while the hot hand will sense it as cold.

PINGBACKS

Pingback: Thermal expansion of liquids and solids