THERMAL EXPANSION OF LIQUIDS AND SOLIDS

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References: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.7 - 1.8

The volume thermal expansion coefficient of a substance as its temperature is increased at constant pressure is defined as the fractional change in volume per degree kelvin, that is

$$\beta \equiv \frac{\Delta V/V}{\Delta T} \tag{1}$$

Example 1. For mercury, $\beta = 1.80 \times 10^{-4}\ \mathrm{K}^{-1}$ so if we have a typical mercury thermometer with a cylindrical bulb $h = 1$ cm long and with a radius of $r = 0.2$ cm, and the scale on the thermometer is 1 mm per degree, then we can work out the inside diameter 2ρ of the tube. We get

$$\begin{aligned} V &= \pi r^2 h = 1.26 \times 10^{-7}\ \mathrm{m}^3 && (2)\\ \Delta V &= \beta V \Delta T && (3)\\ &= \left(1.80 \times 10^{-4}\right)\left(1.26 \times 10^{-7}\right)(1) && (4)\\ &= 2.27 \times 10^{-11}\ \mathrm{m}^3 && (5)\\ \rho &= \sqrt{\frac{\Delta V}{\pi\left(10^{-3}\ \mathrm{m}\right)}} && (6)\\ &= 8.5 \times 10^{-5}\ \mathrm{m} && (7) \end{aligned}$$

The diameter is therefore about 0.2 mm.

Example 2. For water, β varies a lot in the liquid region. At 100°C, it is $7.5 \times 10^{-4}\ \mathrm{K}^{-1}$ and decreases to zero at 4° C. Between the freezing point at 0°C and 4° C, β is actually negative, with its largest negative value of $\beta = -0.68 \times 10^{-4}\ \mathrm{K}^{-1}$ at the freezing point. That is, melting ice actually contracts (becomes denser) as its temperature increases to 4° C which is the reason that ice floats. If β were positive over the entire liquid range of water, a cooling lake would start to freeze from the bottom up rather than from the top down as it does in nature.

Incidentally, you might think that because $\beta > 0$ for temperatures between 4° C and 100° C, ice might sink in hot water (before it melts, of course). However, at standard pressure, the density of boiling water is 0.9584 g cm^{-3} while the density of ice at 0° C is 0.9167 g cm^{-3} so ice floats even in boiling water.

For solids, we can define a linear thermal expansion coefficient as the fractional change of length per degree of increase in temperature:

$$\alpha \equiv \frac{\Delta L/L}{\Delta T} \tag{8}$$

Example 3. For steel, $\alpha = 1.1 \times 10^{-5} \text{ K}^{-1}$. Assuming this value is constant over the range of outdoor air temperatures, we can estimate the change in length of a 1 km steel bridge between winter and summer. In Dundee, the temperature doesn't vary as much as in more continental locations, but we'll take a cold day in Dundee to be 0° C and a hot day to be 25° C, so $\Delta T = 25$. The change in length is therefore

$$\Delta L = \left(1.1 \times 10^{-5}\right)(25)\left(10^3\right) = 0.275 \text{ m} \tag{9}$$

Thus the change in length is far from negligible, which is the reason why long bridges are built in sections with expansion joints in between.

Example 4. One type of thermometer consists of a spiral consisting of two different metal strips (with different values of α) bonded together. Since the metals expand at different rates, the spiral winds and unwinds as the temperature changes. A dial can be attached to the end of the spiral to measure its position and thus give a measure of temperature.

Example 5. If a solid is not isotropic, it has different values of α in each direction, so in rectangular coordinates we have α_x, α_y and α_z defined as $\Delta x/(x\Delta T)$ and so on. For a rectangular solid we have

$$\begin{aligned}
V &= xyz \qquad &(10)\\
\Delta V &= (x+\Delta x)(y+\Delta y)(z+\Delta z) - xyz \qquad &(11)\\
&= yz\Delta x + xz\Delta y + xy\Delta z + \mathcal{O}\left(\Delta x^2\right) \qquad &(12)\\
\frac{\Delta V}{V} &= \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z} + \mathcal{O}\left(\Delta x^2\right) \qquad &(13)\\
\frac{\Delta V}{V\Delta T} &= \alpha_x + \alpha_y + \alpha_z + \mathcal{O}\left(\Delta x^2\right) \qquad &(14)\\
&= \beta \qquad &(15)
\end{aligned}$$

Thus to first order in changes in length

$$\beta = \alpha_x + \alpha_y + \alpha_z \tag{16}$$

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