

## THERMAL EXPANSION OF LIQUIDS AND SOLIDS

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References: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.7 - 1.8

The volume thermal expansion coefficient of a substance as its temperature is increased at constant pressure is defined as the fractional change in volume per degree kelvin, that is

$$\beta \equiv \frac{\Delta V/V}{\Delta T} \quad (1)$$

**Example 1.** For mercury,  $\beta = 1.80 \times 10^{-4} \text{ K}^{-1}$  so if we have a typical mercury thermometer with a cylindrical bulb  $h = 1 \text{ cm}$  long and with a radius of  $r = 0.2 \text{ cm}$ , and the scale on the thermometer is 1 mm per degree, then we can work out the inside diameter  $2\rho$  of the tube. We get

$$V = \pi r^2 h = 1.26 \times 10^{-7} \text{ m}^3 \quad (2)$$

$$\Delta V = \beta V \Delta T \quad (3)$$

$$= (1.80 \times 10^{-4}) (1.26 \times 10^{-7}) (1) \quad (4)$$

$$= 2.27 \times 10^{-11} \text{ m}^3 \quad (5)$$

$$\rho = \sqrt{\frac{\Delta V}{\pi (10^{-3} \text{ m})}} \quad (6)$$

$$= 8.5 \times 10^{-5} \text{ m} \quad (7)$$

The diameter is therefore about 0.2 mm.

**Example 2.** For water,  $\beta$  varies a lot in the liquid region. At  $100^\circ\text{C}$ , it is  $7.5 \times 10^{-4} \text{ K}^{-1}$  and decreases to zero at  $4^\circ\text{C}$ . Between the freezing point at  $0^\circ\text{C}$  and  $4^\circ\text{C}$ ,  $\beta$  is actually negative, with its largest negative value of  $\beta = -0.68 \times 10^{-4} \text{ K}^{-1}$  at the freezing point. That is, melting ice actually contracts (becomes denser) as its temperature increases to  $4^\circ\text{C}$  which is the reason that ice floats. If  $\beta$  were positive over the entire liquid range of water, a cooling lake would start to freeze from the bottom up rather than from the top down as it does in nature.

Incidentally, you might think that because  $\beta > 0$  for temperatures between  $4^\circ\text{C}$  and  $100^\circ\text{C}$ , ice might sink in hot water (before it melts, of

course). However, at standard pressure, the density of boiling water is  $0.9584 \text{ g cm}^{-3}$  while the density of ice at  $0^\circ \text{ C}$  is  $0.9167 \text{ g cm}^{-3}$  so ice floats even in boiling water.

For solids, we can define a linear thermal expansion coefficient as the fractional change of length per degree of increase in temperature:

$$\alpha \equiv \frac{\Delta L/L}{\Delta T} \quad (8)$$

**Example 3.** For steel,  $\alpha = 1.1 \times 10^{-5} \text{ K}^{-1}$ . Assuming this value is constant over the range of outdoor air temperatures, we can estimate the change in length of a 1 km steel bridge between winter and summer. In Dundee, the temperature doesn't vary as much as in more continental locations, but we'll take a cold day in Dundee to be  $0^\circ \text{ C}$  and a hot day to be  $25^\circ \text{ C}$ , so  $\Delta T = 25$ . The change in length is therefore

$$\Delta L = (1.1 \times 10^{-5}) (25) (10^3) = 0.275 \text{ m} \quad (9)$$

Thus the change in length is far from negligible, which is the reason why long bridges are built in sections with expansion joints in between.

**Example 4.** One type of thermometer consists of a spiral consisting of two different metal strips (with different values of  $\alpha$ ) bonded together. Since the metals expand at different rates, the spiral winds and unwinds as the temperature changes. A dial can be attached to the end of the spiral to measure its position and thus give a measure of temperature.

**Example 5.** If a solid is not isotropic, it has different values of  $\alpha$  in each direction, so in rectangular coordinates we have  $\alpha_x$ ,  $\alpha_y$  and  $\alpha_z$  defined as  $\Delta x/(x\Delta T)$  and so on. For a rectangular solid we have

$$V = xyz \quad (10)$$

$$\Delta V = (x + \Delta x)(y + \Delta y)(z + \Delta z) - xyz \quad (11)$$

$$= yz\Delta x + xz\Delta y + xy\Delta z + \mathcal{O}(\Delta x^2) \quad (12)$$

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z} + \mathcal{O}(\Delta x^2) \quad (13)$$

$$\frac{\Delta V}{V\Delta T} = \alpha_x + \alpha_y + \alpha_z + \mathcal{O}(\Delta x^2) \quad (14)$$

$$= \beta \quad (15)$$

Thus to first order in changes in length

$$\beta = \alpha_x + \alpha_y + \alpha_z \quad (16)$$

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