IDEAL GAS LAW

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References: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.9 - 1.15

The *ideal gas law* was originally stated as an experimental result and is

$$PV = nRT \tag{1}$$

where P is the pressure, V is the volume, n is the number of moles of the gas, T is the temperature in kelvins and R is the gas constant.

- Pressure is force per unit area so its SI unit is $N\,m^{-2}$, otherwise known as the Pascal (Pa). The Pascal is quite a small pressure; it's roughly equivalent to the pressure exerted by a sheet of paper lying flat on a desk. Another common unit of pressure is the *atmosphere* which is the mean pressure exerted by Earth's atmosphere at sea level. One atmosphere is 9.80665×10^4 Pa.
- Volume in SI is measured in m^3.
- A *mole* is defined as the number of carbon atoms in 12 grams of carbon-12, and is equal to Avogadro's number: $n = 6.02 \times 10^{23}$.
- The gas constant is measured experimentally to be $R = 8.31$ J $(mol\,K)^{-1}$.
- The temperature must be measured in kelvins if we're using SI units. In any case, T would have to be in units where $T = 0$ at absolute zero in order to avoid negative quantities in the equation.

If we deal with the actual number of molecules N rather than the number of moles, then the ideal gas law is written as

$$PV = NkT \tag{2}$$

where k is *Boltzmann's constant* and has the value

$$k = \frac{R}{6.02 \times 10^{23}} = 1.381 \times 10^{-23} \text{ J K}^{-1} \tag{3}$$

An ideal gas is a gas in which the molecules are point objects that do not interact with each other so it's an approximation for any real gas. However, if the pressure is low enough so that the average distance between molecules is many times the size of a molecule, the law holds reasonably well.

We can use the law to work out some quantities in everyday situations.

Example 1. How much space does a mole of air occupy at room temperature (293 K = 20 C) and 1 atm pressure?

$$V = \frac{nRT}{P} = \frac{(1)(8.31)(293)}{9.80665 \times 10^4} = 0.0248 \text{ m}^3$$

This is a cube 29 cm on a side so a mole of air is found in roughly a cubic foot.

Example 2. Using the previous example, we can estimate the number of air molecules in a typical room in a house. The room in which I'm typing this blog is about 4 metres square by 3 metres high so its volume is 48 m^3 so the number of air molecules is around

$$N = \frac{48}{0.0248} \times 6.02 \times 10^{23} = 1.16 \times 10^{27} \tag{4}$$

Example 3. Suppose we have two rooms A and B that are the same size and connected by an open doorway, so the air has the same pressure and volume in both rooms. This means that

$$P_A V_A = P_B V_B \tag{5}$$
$$n_A T_A = n_B T_B \tag{6}$$

If room A is warmer than room B then $n_A < n_B$ so room B contains the greater mass of air.

Example 4. From example 1 we can calculate the average distance between molecules in an ideal gas. The volume per molecule is

$$v = \frac{V}{6.02 \times 10^{23}} = 4.124 \times 10^{-26} \text{ m}^3 \tag{7}$$

The cube root of this provides an estimate of the inter-molecule distance:

$$d = 3.455 \times 10^{-9} \text{ m} \tag{8}$$

The size of a water molecule is about 3×10^{-10} m so in a gas of water vapour, the distance is about ten times the molecular size.

Example 5. Roughly speaking, the *relative atomic mass* (previously known as the atomic weight) of an element is the mass of one mole of that element where the various isotopes of the element in the sample occur in the ratios they are found in nature (on Earth, at least). For a few substances, the mass of 1 mole is shown:

Substance	Formula	Mass of 1 mole (g)
Water	H_2O	18.01528
Nitrogen	N_2	28.01348
Lead	Pb	207.2
Quartz	SiO_2	60.0843
Oxygen	O_2	31.9988
Argon	Ar	39.948

Example 6. Using the values in example 5 we can work out the mass of a mole of dry air, which consists of 78% nitrogen, 21% oxygen and 1% argon.

$$m = 0.78 \times 28.01348 + 0.21 \times 31.9988 + 0.01 \times 39.948 = 28.9697 \text{ g} \quad (9)$$

From example 1, this works out to $0.0289697 \div 0.0248 = 1.168$ kg m^{-3} at room temperature and 1 atm pressure, which agrees roughly with values measured by my weather station.

Example 7. In a hot-air balloon, the buoyancy of the hot air relative to the colder air outside must balance the gravitational force on the balloon and its attached basket. To estimate the temperature of the air inside the balloon, we'll need estimates of the mass of the unfilled ballon and basket and of the volume of the filled balloon. We'll take the mass to be 500 kg and the volume to be 2800 m^3 (typical for a balloon that can take 4 or 5 passengers). If the pressure inside equals the pressure outside (reasonable, since the balloon isn't sealed) then the difference between the mass of hot air within the balloon and the mass of the cooler air it displaces must be 500 kg, so

$$V\left(\rho_c - \rho_h\right) = 500 \quad (10)$$

where $\rho_{c,h}$ are the densities of the cold and hot air. If the outdoor temperature is 20 C (293 K), then from example 6

$$\rho_c = 1.168 \text{ kg m}^{-3} \quad (11)$$

$$\rho_h = \rho_c - \frac{500}{V} \quad (12)$$

$$= 1.168 - \frac{500}{2800} \quad (13)$$

$$= 0.9896 \text{ kg m}^{-3} \quad (14)$$

The ratio of the densities is equal to the ratio in the numbers of molecules so

$$\frac{P_c V_c}{P_h V_h} = 1 = \frac{n_c T_c}{n_h T_h} = \frac{\rho_c T_c}{\rho_h T_h} \tag{15}$$

$$T_h = \frac{\rho_c}{\rho_h} T_c \tag{16}$$

$$= \frac{1.168}{0.9896} 293 \tag{17}$$

$$= 345 \text{ K} = 72°\text{C} \tag{18}$$

The mass of air inside the balloon is

$$m = \rho_h V = 2770 \text{ kg} \tag{19}$$

The larger the balloon, the lower the temperature required to generate the required buoyancy.

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