BAROMETRIC EQUATION: THE EXPONENTIAL ATMOSPHERE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.16.

A model for the pressure in the atmosphere as a function of height can be estimated from the ideal gas law. Suppose we have a horizontal slab of air with thickness dz and a unit cross sectional area. The density of air is a function $\rho(z)$ of height z above sea level. The pressure of the air at height z must be equal to the pressure of the air above it plus the weight of the air in the slab. In other words, the *change* in pressure as we go from height z to $z+dz$ is just the weight of the air in the slab of thickness dz, so since pressure is force per unit area

$$dP = -\rho g\, dz \tag{0.1}$$

where g is the acceleration of gravity. The minus sign accounts for the fact that pressure decreases as we go higher.

The density of air can be written using the ideal gas law as

$$\rho = \frac{Nm}{V} \tag{0.2}$$

$$= \frac{m}{kT}P \tag{0.3}$$

where m is the average mass of an air molecule, N is the number of molecules in volume V and T is the temperature. We therefore get a differential equation for the pressure:

$$\frac{dP}{dz} = -\rho g \tag{0.4}$$

$$= -\frac{mg}{kT}P \tag{0.5}$$

This is called the *barometric equation*. In a realistic model, both g and T would depend on z, although since the atmosphere's thickness isn't really large enough to affect g all that much, we can safely take it to be the usual $g = 9.8\text{ m s}^{-2}$. The temperature does decrease substantially as we get higher, but if we want a crude model we can take it to be roughly constant. In that case, we can integrate the equation to get

$$\frac{dP}{P} = -\frac{mg}{kT}dz \tag{0.6}$$

$$P = P(0)\,e^{-mgz/kT} \tag{0.7}$$

so the pressure decreases exponentially with altitude. Further, from 0.3, we have

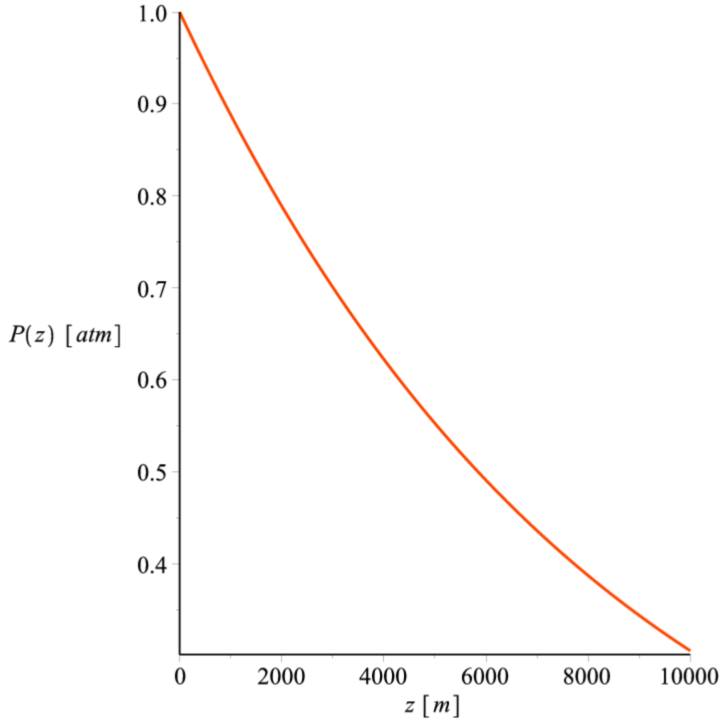
$$\rho(z) = \frac{m}{kT}P(z) = \frac{m}{kT}P(0)\,e^{-mgz/kT} \equiv \rho(0)\,e^{-mgz/kT} \tag{0.8}$$

Assuming a temperature of 288 K (15° C), and taking $P(0) = 1$ atm, we get the following values for pressure at various altitudes:

Altitude z (m)	$P(z)$ (atm)
1430	0.844
3090	0.693
4420	0.592
8850	0.350

The last value is the altitude at the top of Mount Everest and compares with the measured value of 0.333 atm (33.7 kilopascals), so the model isn't too bad.

A graph of pressure versus altitude is as follows:



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