EFFUSION: GAS LEAKING THROUGH A SMALL HOLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.21 - 1.22.

Here are a few examples of calculating the pressure on a surface due to the collisions of a number of molecules (or other small objects) with that surface.

Example 1. For a macroscopic example, suppose there's a hailstorm that sends hailstones of mass 2 g (actually, these would be very big hailstones: larger than 2 cm^3. They'd probably break the window, but never mind) with a speed of 15 m/s at a 45° angle into a window at a rate of $N = 30$ per second. What average pressure do they exert on the window?

The horizontal velocity component is

$$v_x = 15\cos 45^\circ = \frac{15}{\sqrt{2}}\text{m s}^{-1} \tag{1}$$

The horizontal momentum is reversed in each collision (assuming the collisions are elastic), so in one second, the total force exerted on the window is

$$\Delta F = 2mv_xN = 2\times 0.002\times\frac{15}{\sqrt{2}}\times 30 = 1.275\text{ N} \tag{2}$$

The pressure on a window of area $A = 0.5$ m^2 is therefore

$$P = \frac{\Delta F}{A} = 2.55\text{ N m}^{-2} \tag{3}$$

This is much smaller than atmospheric pressure, which is around 10^5 N m^{-2}. Atmospheric pressure doesn't break windows, of course, because it's the same on both sides of the glass.

If there is a pressure difference on either side of a small hole, gas will leak out of the hole in a process called *effusion*. We can get an estimate of the time taken for gas to leak out of a hole using similar ideas to those above.

Suppose we have a container of volume V that contains gas at initial pressure P and temperature T. A small hole of area A is made in the container. Suppose that the average x component of the velocity of those molecules that are moving towards the hole is $\overline{v_x}$. Assuming elastic collisions, each molecule has its x momentum of $m\overline{v_x}$ reversed, so in time Δt, the momentum change per unit time as a result of collisions with an area A is the force, so

$$F = \frac{\Delta N\left(2m\overline{v_x}\right)}{\Delta t} = PA \tag{4}$$

$$\Delta N = \frac{PA\Delta t}{2m\overline{v_x}} \tag{5}$$

We've seen from kinetic energy arguments that the mean square of v_x is

$$\overline{v_x^2} = \frac{kT}{m} \tag{6}$$

so we can use the rms of v_x as an estimate for $\overline{v_x}$:

$$\overline{v_x} \approx \sqrt{\overline{v_x^2}} = \sqrt{\frac{kT}{m}} \tag{7}$$

If the area A is now made into the hole, then assuming that there is a vacuum on the other side of the hole, all the molecules that would have collided with the hole will now escape from the container, and no molecules will pass into the container. In that case, the change ΔN in the number of molecules in the container over time interval Δt is (using the ideal gas law; the minus sign is because N decreases)

$$\frac{\Delta N}{\Delta t} = -\frac{PA}{2m\overline{v_x}} \tag{8}$$

$$\approx -\frac{PA}{2m}\sqrt{\frac{m}{kT}} \tag{9}$$

$$= -\frac{NkTA}{2mV}\sqrt{\frac{m}{kT}} \tag{10}$$

$$= -\frac{A}{2V}\sqrt{\frac{kT}{m}}N \tag{11}$$

Treating this as a differential equation, we can solve it to get

$$N(t) = N(0)\, e^{-At\sqrt{kT}/2V\sqrt{m}} \tag{12}$$

so the characteristic time (after which N drops to $\frac{1}{e}$ of its initial value) is

$$\tau = \frac{2V}{A}\sqrt{\frac{m}{kT}} \tag{13}$$

Note that τ is independent of pressure, which may seem odd, since we would think that if the pressure inside the container is higher, the gas would escape faster. It is true that for a higher pressure, more gas will escape in a given time, but the *fraction* of the initial quantity of gas that escapes in a given time will be the same, regardless of the pressure.

Example 2. Suppose we have a 1 litre container at room temperature (in a vacuum) punctured by a 1 mm^2 hole. The average mass of an air molecule is

$$m = \frac{28.9697 \times 10^{-3}}{6.02 \times 10^{23}} = 4.81 \times 10^{-26} \text{ kg} \tag{14}$$

The characteristic time is therefore

$$\tau = \frac{2 \times 10^{-3}}{10^{-6}} \sqrt{\frac{4.81 \times 10^{-26}}{1.38 \times 10^{-23} \times 293}} = 6.9 \text{ s} \tag{15}$$

Example 3. Puncture in a bicycle tire. A typical bicycle tire has a diameter of around 0.63 m and a cross-sectional diameter of about 0.03 m, so its volume is around

$$V = 2\pi \frac{0.63}{2} \times \pi \left(\frac{0.03}{2}\right)^2 = 1.4 \times 10^{-3} \text{ m}^3 = 1.4 \text{ litres} \tag{16}$$

Suppose the tire goes flat after 1 hour, so that $\tau = 3600$ s. How big is the hole?

$$A = \frac{2V}{\tau} \sqrt{\frac{m}{kT}} \tag{17}$$

$$= \frac{2 \times 1.4 \times 10^{-3}}{3600} \sqrt{\frac{4.81 \times 10^{-26}}{1.38 \times 10^{-23} \times 293}} \tag{18}$$

$$= 2.68 \times 10^{-9} \text{ m}^2 \tag{19}$$

The hole has a diameter of

$$d = 2\sqrt{\frac{A}{\pi}} = 0.058 \text{ mm} \tag{20}$$

Example 4. Schroeder's final example cites an episode from Jules Verne's novel *Around the Moon*, in which space travelers eject a dead dog by opening a porthole in the spaceship, throwing the dog out and quickly closing the window again. This seems risky, but how much air would they actually lose in doing so?

Suppose it's a small dog, so the porthole has a diameter of 30 cm, and the spaceship has a volume of $V = 100\ \mathrm{m}^3$ (about the size of a smallish bedroom). The crew is efficient, so they can open the window, toss out the dog and close the window within 1 second. We get

$$A = \pi\left(\frac{0.3}{2}\right)^2 = 0.071\ \mathrm{m}^2 \tag{21}$$

$$\tau = \frac{2\times 100}{0.071}\sqrt{\frac{4.81\times 10^{-26}}{1.38\times 10^{-23}\times 293}} \tag{22}$$

$$= 9.72\ \mathrm{s} \tag{23}$$

With $t = 1$ s, they would retain a fraction of their air given by

$$\frac{N(1)}{N(0)} = e^{-1/9.72} = 0.9 \tag{24}$$

so they'd lose about 10% of their air. They'd probably survive, but I think I'd rather use an airlock.