COMPRESSION WORK

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.31 - 1.32.

The work done when compressing a gas (or any substance) by, for example, pushing a piston with area A in by a distance Δx is the force F times the distance moved Δx. If the gas exerts a pressure P on the piston, then the force required to compress the gas is $F = PA$, and the work done is

$$W = PA\Delta x \tag{1}$$

If we move the piston slowly enough that the gas has a chance to respond to the volume change by equalizing its pressure throughout the volume, then we can assume that at any given instant of time, the pressure is uniform throughout the gas. Compressing a gas will, of course, usually change the pressure, so we can think of the pressure as a function of the volume V. Since moving the piston a distance Δx causes a change in volume $-\Delta V$ (negative because the volume decreases), we have in general that

$$W = -P(V)\Delta V \tag{2}$$

A process that happens slowly enough to allow the system to come to equilibrium after each infinitestimal change is called *quasistatic*. When the piston moves, the effect travels through the gas as a shock wave, which travels at the speed of sound, so usually if we move the piston much more slowly than the speed of sound in the gas, the process is quasistatic.

The total work done in compressing a gas from some initial volume V_i to a final volume V_f is therefore

$$W = -\int_{V_i}^{V_f} P(V)\,dV \tag{3}$$

Example 1. We have some helium (a monatomic gas) at an initial volume of 1 litre (so $V_i = 10^{-3}\ \mathrm{m}^3$) and pressure of 1 atm ($P(V_i) = 1.013\times 10^5$ Pa). The helium expands to a final volume of 3 litres in such a way that its pressure increases in direct proportion to its volume. Assuming that the mass of helium remains constant, this could be done, for example, by heating the helium as it expands. The pressure is given by

$$P(V) = AV \tag{4}$$

where A is a constant with value

$$A = 1.013 \times 10^8 \text{ Pa m}^{-3} \tag{5}$$

The graph of P vs. V is a straight line with slope A that passes through the origin. The work done on the gas in the above process is therefore

$$\begin{aligned}
W &= -1.013 \times 10^8 \int_{10^{-3}}^{3\times 10^{-3}} V\,dV && (6)\\
&= -1.013 \times 10^8 \left(\frac{1}{2}V^2\right)\Bigg|_{10^{-3}}^{3\times 10^{-3}} && (7)\\
&= -405.2 \text{ J} && (8)
\end{aligned}$$

The fact that W is negative shows that the gas does work on its surroundings.

The thermal energy of a monatomic gas is $\frac{3}{2}NkT = \frac{3}{2}PV$ so the change in thermal energy due to the expansion is

$$\begin{aligned}
\Delta U &= \frac{3}{2}\left[P\left(V_f\right)V_f - P\left(V_i\right)V_i\right] && (9)\\
&= \frac{3}{2}\left(3 \times 1.013 \times 10^5 \times 3 \times 10^{-3} - 1.013 \times 10^5 \times 10^{-3}\right) && (10)\\
&= 1215.6 \text{ J} && (11)
\end{aligned}$$

The heat added to the gas must be enough to do the work plus increase the thermal energy, so

$$Q = \Delta U - W = 1620.8 \text{ J} \tag{12}$$

Example 2. Liquid water is difficult to compress, but at a pressure of 200 atm its volume is reduced to 99% of its value at atmospheric pressure. If we assume that the reduction in volume is linear with the increase in pressure, then

$$P(V) = AV + B \tag{13}$$

where A and B are constants. At $P = 1 \text{ atm} = 1.013 \times 10^5$ Pa, $V = V_0$ and at $P = 200 \text{ atm} = 2.026 \times 10^7$ Pa, $V = 0.99V_0$, so

$$A = -\frac{2.016\times 10^{9}}{V_0}\text{Pa m}^{-3} \tag{14}$$

$$B = 2.016\times 10^{9}\text{ Pa} \tag{15}$$

The work required to compress 1 litre of water to 99% can be found from

$$A = -\frac{2.016\times 10^{9}}{10^{-3}} = -2.016\times 10^{12}\text{ Pa m}^{-3} \tag{16}$$

$$W = 2.016\times 10^{12}\int_{10^{-3}}^{0.99\times 10^{-3}} V\,dV + 2.016\times 10^{9}\left(0.01\times 10^{-3}\right) \tag{17}$$

$$= 101\text{ J} \tag{18}$$

Note that if we assume a constant pressure of 200 atm doing the compressing, we get

$$W = P\Delta V \tag{19}$$

$$= 2.026\times 10^{7}\times 0.01\times 10^{-3} \tag{20}$$

$$= 202.6\text{ J} \tag{21}$$

or roughly twice as much. This is surprisingly small considering the large pressure that's needed, but then work is the pressure times the change in volume, and the latter is very small.

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