PV DIAGRAMS: A MONATOMIC IDEAL GAS Follows A TRIANGULAR CYCLE

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We can plot the state of an ideal gas on a plot of pressure versus volume (a PV diagram). Using this diagram we can work out a few facts about how much heat and work flows into or out of the gas.

As an example, a monatomic ideal gas follows a triangular cycle on a PV diagram, starting at pressure \( P_1 \) and volume \( V_1 \). On the first leg (side A) of the triangle, the pressure is held constant while the volume increases to \( V_2 \) (so the path is a horizontal line). Then (side B) the volume is held constant and the pressure is increased to \( P_2 \), giving a vertical line on the PV diagram. Finally the pressure is reduced back to \( P_1 \) and volume back to \( V_1 \) along side C, which is a straight, diagonal line with slope \( \frac{P_2 - P_1}{V_2 - V_1} \).

In a compression (or expansion) problem, the work done on the gas is

\[
W = - \int_{V_i}^{V_f} P(V) \, dV \tag{1}
\]

For this problem, the work done on side A is

\[
W_A = -P_1 \, (V_2 - V_1) < 0 \tag{2}
\]

On side B (since \( V \) is constant)

\[
W_B = 0 \tag{3}
\]

On side C, the work done is the negative of that done on side A, plus the area of the (right-angled) triangle, so

\[
W_C = P_1 \, (V_2 - V_1) + \frac{1}{2} \, (V_2 - V_1) \, (P_2 - P_1) > 0 \tag{4}
\]

The total work done on the gas is

\[
W = W_A + W_B + W_C = \frac{1}{2} \, (V_2 - V_1) \, (P_2 - P_1) > 0 \tag{5}
\]

That is, the total work is just the area of the triangle.

From the equipartition theorem, the thermal energy of the gas is
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\[ U = \frac{3}{2} N k T = \frac{3}{2} PV \]  
(6)

so along side A (since \( P \) is constant and \( V \) increases)

\[ \Delta U_A = \frac{3}{2} P_1 (V_2 - V_1) > 0 \]  
(7)

along side B

\[ \Delta U_B = \frac{3}{2} V_2 (P_2 - P_1) > 0 \]  
(8)

(since \( V \) is constant and \( P \) increases) and along side C

\[ \Delta U_C = -\frac{3}{2} (P_2 V_2 - P_1 V_1) < 0 \]  
(9)

(since both \( P \) and \( V \) decrease). The net change in \( U \) after going round all three sides is zero, since the gas is back in its original state.

From conservation of energy, we can get the heat \( Q = \Delta U - W \) on each side. On side A

\[ Q_A = \frac{3}{2} P_1 (V_2 - V_1) + P_1 (V_2 - V_1) \]  
(10)

\[ = \frac{5}{2} P_1 (V_2 - V_1) > 0 \]  
(11)

On side B

\[ Q_B = \frac{3}{2} V_2 (P_2 - P_1) + 0 > 0 \]  
(12)

And on side C

\[ Q_C = -\frac{3}{2} (P_2 V_2 - P_1 V_1) - P_1 (V_2 - V_1) - \frac{1}{2} (V_2 - V_1) (P_2 - P_1) < 0 \]  
(13)

The total heat added to the gas is

\[ Q = Q_A + Q_B + Q_C = -\frac{1}{2} (V_2 - V_1) (P_2 - P_1) = -W < 0 \]  
(14)

Since this is negative, a net amount of heat is emitted by the process. Thus the overall process converts the net work done on the gas to heat.

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