PV DIAGRAMS: A MONATOMIC IDEAL GAS FOLLOWS A TRIANGULAR CYCLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.33.

We can plot the state of an ideal gas on a plot of pressure versus volume (a PV diagram). Using this diagram we can work out a few facts about how much heat heat and work flows into or out of the gas.

As an example, a monatomic ideal gas follows a triangular cycle on a PV diagram, starting at pressure P_1 and volume V_1. On the first leg (side A) of the triangle, the pressure is held constant while the volume increases to V_2 (so the path is a horizontal line). Then (side B) the volume is held constant and the pressure is increased to P_2, giving a vertical line on the PV diagram. Finally the pressure is reduced back to P_1 and volume back to V_1 along side C, which is a straight, diagonal line with slope $(P_2 - P_1)/(V_2 - V_1)$.

In a compression (or expansion) problem, the work done on the gas is

$$W = -\int_{V_i}^{V_f} P(V)\,dV \tag{0.1}$$

For this problem, the work done on side A is

$$W_A = -P_1(V_2 - V_1) < 0 \tag{0.2}$$

On side B (since V is constant)

$$W_B = 0 \tag{0.3}$$

On side C, the work done is the negative of that done on side A, plus the area of the (right-angled) triangle, so

$$W_C = P_1(V_2 - V_1) + \frac{1}{2}(V_2 - V_1)(P_2 - P_1) > 0 \tag{0.4}$$

The total work done on the gas is

$$W = W_A + W_B + W_C = \frac{1}{2}(V_2 - V_1)(P_2 - P_1) > 0 \tag{0.5}$$

That is, the total work is just the area of the triangle.

From the equipartition theorem, the thermal energy of the gas is

$$U=\frac{3}{2}NkT=\frac{3}{2}PV \tag{0.6}$$

so along side A (since P is constant and V increases)

$$\Delta U_A=\frac{3}{2}P_1\left(V_2-V_1\right)>0 \tag{0.7}$$

along side B

$$\Delta U_B=\frac{3}{2}V_2\left(P_2-P_1\right)>0 \tag{0.8}$$

(since V is constant and P increases) and along side C

$$\Delta U_C=-\frac{3}{2}\left(P_2V_2-P_1V_1\right)<0 \tag{0.9}$$

(since both P and V decrease). The net change in U after going round all three sides is zero, since the gas is back in its original state.

From conservation of energy, we can get the heat $Q=\Delta U-W$ on each side. On side A

$$Q_A = \frac{3}{2}P_1\left(V_2-V_1\right)+P_1\left(V_2-V_1\right) \tag{0.10}$$

$$= \frac{5}{2}P_1\left(V_2-V_1\right)>0 \tag{0.11}$$

On side B

$$Q_B=\frac{3}{2}V_2\left(P_2-P_1\right)+0>0 \tag{0.12}$$

And on side C

$$Q_C=-\frac{3}{2}\left(P_2V_2-P_1V_1\right)-P_1\left(V_2-V_1\right)-\frac{1}{2}\left(V_2-V_1\right)\left(P_2-P_1\right)<0 \tag{0.13}$$

The total heat added to the gas is

$$Q=Q_A+Q_B+Q_C=-\frac{1}{2}\left(V_2-V_1\right)\left(P_2-P_1\right)=-W<0 \tag{0.14}$$

Since this is negative, a net amount of heat is emitted by the process. Thus the overall process converts the net work done on the gas to heat.

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