

PV DIAGRAMS: A MONATOMIC IDEAL GAS FOLLOWS A TRIANGULAR CYCLE

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.33.

We can plot the state of an ideal gas on a plot of pressure versus volume (a PV diagram). Using this diagram we can work out a few facts about how much heat and work flows into or out of the gas.

As an example, a monatomic ideal gas follows a triangular cycle on a PV diagram, starting at pressure P_1 and volume V_1 . On the first leg (side A) of the triangle, the pressure is held constant while the volume increases to V_2 (so the path is a horizontal line). Then (side B) the volume is held constant and the pressure is increased to P_2 , giving a vertical line on the PV diagram. Finally the pressure is reduced back to P_1 and volume back to V_1 along side C, which is a straight, diagonal line with slope $(P_2 - P_1) / (V_2 - V_1)$.

In a compression (or expansion) problem, the work done on the gas is

$$W = - \int_{V_i}^{V_f} P(V) dV \quad (1)$$

For this problem, the work done on side A is

$$W_A = -P_1 (V_2 - V_1) < 0 \quad (2)$$

On side B (since V is constant)

$$W_B = 0 \quad (3)$$

On side C, the work done is the negative of that done on side A, plus the area of the (right-angled) triangle, so

$$W_C = P_1 (V_2 - V_1) + \frac{1}{2} (V_2 - V_1) (P_2 - P_1) > 0 \quad (4)$$

The total work done on the gas is

$$W = W_A + W_B + W_C = \frac{1}{2} (V_2 - V_1) (P_2 - P_1) > 0 \quad (5)$$

That is, the total work is just the area of the triangle.

From the equipartition theorem, the thermal energy of the gas is

$$U = \frac{3}{2}NkT = \frac{3}{2}PV \quad (6)$$

so along side A (since P is constant and V increases)

$$\Delta U_A = \frac{3}{2}P_1(V_2 - V_1) > 0 \quad (7)$$

along side B

$$\Delta U_B = \frac{3}{2}V_2(P_2 - P_1) > 0 \quad (8)$$

(since V is constant and P increases) and along side C

$$\Delta U_C = -\frac{3}{2}(P_2V_2 - P_1V_1) < 0 \quad (9)$$

(since both P and V decrease). The net change in U after going round all three sides is zero, since the gas is back in its original state.

From conservation of energy, we can get the heat $Q = \Delta U - W$ on each side. On side A

$$Q_A = \frac{3}{2}P_1(V_2 - V_1) + P_1(V_2 - V_1) \quad (10)$$

$$= \frac{5}{2}P_1(V_2 - V_1) > 0 \quad (11)$$

On side B

$$Q_B = \frac{3}{2}V_2(P_2 - P_1) + 0 > 0 \quad (12)$$

And on side C

$$Q_C = -\frac{3}{2}(P_2V_2 - P_1V_1) - P_1(V_2 - V_1) - \frac{1}{2}(V_2 - V_1)(P_2 - P_1) < 0 \quad (13)$$

The total heat added to the gas is

$$Q = Q_A + Q_B + Q_C = -\frac{1}{2}(V_2 - V_1)(P_2 - P_1) = -W < 0 \quad (14)$$

Since this is negative, a net amount of heat is emitted by the process. Thus the overall process converts the net work done on the gas to heat.

PINGBACKS

Pingback: PV diagrams: a diatomic ideal gas undergoes a rectangular cycle

Pingback: Isothermal and adiabatic compression of an ideal gas