

## ISOTHERMAL AND ADIABATIC COMPRESSION OF AN IDEAL GAS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.35 - 1.36.

When we compress an ideal gas, we can do so in a variety of ways. If we do the compression very slowly so that the gas is always in thermal equilibrium with its environment, then the temperature stays constant and the compression is *isothermal*.

On the other hand, if we compress the gas quickly so that it doesn't have a chance to exchange heat with its environment, the temperature will change. This sort of compression is called *adiabatic*. [Incidentally, this use of the term 'adiabatic' has nothing to do with an adiabatic process in quantum mechanics and, in fact, the meaning is almost the opposite. In thermodynamics, an adiabatic compression occurs quickly so that heat doesn't have a chance to escape; in quantum mechanics, an adiabatic process is one in which the Hamiltonian changes slowly. I'm not sure why the term is used in quantum mechanics as it's derived the Greek *adiabatos*, which means 'impassable', so its use in thermodynamics as a process that is impassable to heat makes sense.]

Schroeder works out the details for these two types of compression in his section 1.5, so I'll just review the main ideas here.

**Isothermal compression.** For isothermal compression, the temperature is constant so from the ideal gas law

$$(1) \quad P = \frac{NkT}{V} = \frac{\text{constant}}{V}$$

The plot of  $P$  versus  $V$  on a PV diagram is a hyperbola known as an *isotherm*.

Since the thermal energy of a gas whose molecules have  $f$  degrees of freedom is given by

$$(2) \quad U = \frac{f}{2}NkT$$

the thermal energy of an isothermal process doesn't change, so the work required to compress the gas must be entirely converted into the heat lost

during the isothermal process. If the gas is compressed from volume  $V_i$  to  $V_f$ , the work and heat are

$$(3) \quad Q = -W = NkT \ln \frac{V_f}{V_i}$$

If the gas is being compressed,  $V_f < V_i$ ,  $Q < 0$  (meaning that the gas loses heat) and  $W > 0$  (work is done *on* the gas).

**Adiabatic compression.** Even though adiabatic compression must be done quickly to prevent heat loss, it can still be done slowly compared to the speed of sound in the gas, so that the quasistatic assumption (that the pressure of the gas is always uniform throughout the volume) is valid. For example, if you pump up a bicycle tire, you'll find that the pump gets hot, but the speed at which you push the pump is significantly less than the speed of sound, so the compression is, to a good approximation, quasistatic.

By combining the equipartition theorem with the ideal gas law (details in Schroeder section 1.5) we arrive at a differential equation relating  $T$  and  $V$ :

$$(4) \quad \frac{f}{2} \frac{dT}{T} = -\frac{dV}{V}$$

At a given temperature and volume, if  $V$  is decreased (the gas is compressed), the temperature rises. Integrating this equation gives the relation

$$(5) \quad VT^{f/2} = K$$

where  $K$  is a constant.

We can get a relation between  $P$  and  $V$  by using the ideal gas law in this equation.

$$(6) \quad T = \frac{PV}{Nk}$$

$$(7) \quad VT^{f/2} = \frac{V^{1+f/2} P^{f/2}}{(Nk)^{f/2}} = K$$

$$(8) \quad V^{(2+f)/2} P^{f/2} = K (Nk)^{f/2}$$

$$(9) \quad V^{(2+f)/f} P = K^{2/f} Nk$$

or, defining the *adiabatic exponent*  $\gamma \equiv (2+f)/f$ , we have the relation for a quasistatic adiabatic process:

$$(10) \quad \boxed{V^\gamma P = \text{constant}}$$

**Example.** Suppose we compress a litre of air (assumed to be an ideal gas of diatomic molecules, with 5 degrees of freedom) at atmospheric pressure to a pressure of 7 atm. If the process is adiabatic, then the final volume is found from

$$(11) \quad \gamma = \frac{7}{5}$$

$$(12) \quad P_i V_i^{7/5} = 1 \text{ atm litre}^{7/5}$$

$$(13) \quad P_f V_f^{7/5} = 7 V_f^{7/5}$$

$$(14) \quad V_f = \frac{1}{7^{5/7}} = 0.249 \text{ litre}$$

The work done in the compression is

$$(15) \quad W = - \int_{V_i}^{V_f} P(V) dV$$

$$(16) \quad = - \int_1^{0.249} \frac{dV}{V^{7/5}}$$

$$(17) \quad = \frac{5}{2} \frac{1}{V^{2/5}} \Big|_1^{0.249}$$

$$(18) \quad = 1.86 \text{ atm litre}$$

$$(19) \quad = 1.86 \times 1.013 \times 10^5 \times 10^{-3} \text{ J}$$

$$(20) \quad = 188.4 \text{ J}$$

From 5 we can find the temperature after compression, given that the initial temperature is  $T_i = 300 \text{ K}$ . We have

$$(21) \quad \frac{V_f T_f^{5/2}}{V_i T_i^{5/2}} = 1$$

$$(22) \quad T_f = \left( \frac{V_i}{V_f} \right)^{2/5} T_i$$

$$(23) \quad = \left( \frac{1}{0.249} \right)^{2/5} 300$$

$$(24) \quad = 523 \text{ K}$$

That is very hot; it's approaching the melting point of lead (600 K). This pressure is actually in the range of a road bicycle tire, so it's not surprising that a bicycle pump can get very hot when you're inflating a tire.

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