

## ISOTHERMAL VERSUS ADIABATIC EXPANSION OF AN IDEAL GAS BUBBLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.38.

As a simple example of isothermal versus adiabatic expansion of an ideal gas, suppose that two identical bubbles form at the bottom of a lake. Bubble A rises quickly so that no heat is exchanged with the surrounding water, while bubble B rises slowly (bumping off the leaves of some lakeweed, for example) so that its temperature remains constant (assuming that the lake's water temperature is the same everywhere).

Bubble A experiences adiabatic expansion, so it obeys the relation

$$(1) \quad PV^\gamma = A$$

for some constant  $A$ . Bubble B expands isothermally, so

$$(2) \quad PV = NkT = \text{constant}$$

The initial volumes  $V_0$  and pressures  $P_0$  of the two bubbles are the same so

$$(3) \quad A = P_0 V_0^\gamma$$

$$(4) \quad NkT = P_0 V_0$$

When the bubbles reach the surface of the lake, the pressure has reduced to  $P_1$  so the volumes of the bubbles are

$$(5) \quad V_A = \left(\frac{P_0}{P_1}\right)^{1/\gamma} V_0$$

$$(6) \quad V_B = \frac{P_0}{P_1} V_0$$

Since  $\gamma = (f + 2) / f > 1$  where  $f$  is the number of degrees of freedom, and  $P_0 > P_1$

$$(7) \quad \left(\frac{P_0}{P_1}\right)^{1/\gamma} < \frac{P_0}{P_1}$$

so  $V_B > V_A$ . That is, the bubble that rises slowly will be larger than the bubble that rises quickly when they reach the surface.