ISOTHERMAL VERSUS ADIABATIC EXPANSION OF AN IDEAL GAS BUBBLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.38.

As a simple example of isothermal versus adiabatic expansion of an ideal gas, suppose that two identical bubbles form at the bottom of a lake. Bubble A rises quickly so that no heat is exchanged with the surrounding water, while bubble B rises slowly (bumping off the leaves of some lakeweed, for example) so that its temperature remains constant (assuming that the lake's water temperature is the same everywhere).

Bubble A experiences adiabatic expansion, so it obeys the relation

$$PV^{\gamma}=A \tag{0.1}$$

for some constant A. Bubble B expands isothermally, so

$$PV=NkT=\text{constant} \tag{0.2}$$

The initial volumes V_0 and pressures P_0 of the two bubbles are the same so

$$A = P_0V_0^{\gamma} \tag{0.3}$$

$$NkT = P_0V_0 \tag{0.4}$$

When the bubbles reach the surface of the lake, the pressure has reduced to P_1 so the volumes of the bubbles are

$$V_A = \left(\frac{P_0}{P_1}\right)^{1/\gamma}V_0 \tag{0.5}$$

$$V_B = \frac{P_0}{P_1}V_0 \tag{0.6}$$

Since $\gamma=(f+2)/f>1$ where f is the number of degrees of freedom, and $P_0>P_1$

$$\left(\frac{P_0}{P_1}\right)^{1/\gamma} < \frac{P_0}{P_1} \tag{0.7}$$

so $V_B > V_A$. That is, the the bubble that rises slowly will be larger than the bubble that rises quickly when they reach the surface.