

ISOTHERMAL VERSUS ADIABATIC EXPANSION OF AN IDEAL GAS BUBBLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.38.

As a simple example of isothermal versus adiabatic expansion of an ideal gas, suppose that two identical bubbles form at the bottom of a lake. Bubble A rises quickly so that no heat is exchanged with the surrounding water, while bubble B rises slowly (bumping off the leaves of some lakeweed, for example) so that its temperature remains constant (assuming that the lake's water temperature is the same everywhere).

Bubble A experiences adiabatic expansion, so it obeys the relation

$$PV^\gamma = A \quad (1)$$

for some constant A . Bubble B expands isothermally, so

$$PV = NkT = \text{constant} \quad (2)$$

The initial volumes V_0 and pressures P_0 of the two bubbles are the same so

$$A = P_0 V_0^\gamma \quad (3)$$

$$NkT = P_0 V_0 \quad (4)$$

When the bubbles reach the surface of the lake, the pressure has reduced to P_1 so the volumes of the bubbles are

$$V_A = \left(\frac{P_0}{P_1}\right)^{1/\gamma} V_0 \quad (5)$$

$$V_B = \frac{P_0}{P_1} V_0 \quad (6)$$

Since $\gamma = (f + 2) / f > 1$ where f is the number of degrees of freedom, and $P_0 > P_1$

$$\left(\frac{P_0}{P_1}\right)^{1/\gamma} < \frac{P_0}{P_1} \quad (7)$$

so $V_B > V_A$. That is, the bubble that rises slowly will be larger than the bubble that rises quickly when they reach the surface.