BULK MODULUS AND THE SPEED OF SOUND

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.39.

The *bulk modulus* of a substance is a measure of how compressible that substance is. It compares the fractional change in volume to the pressure applied to create that change. Its definition is

$$B \equiv \frac{\Delta P}{-\Delta V/V} \tag{0.1}$$

where the minus sign in the denominator is there because an increase in pressure causes a decrease in volume.

However, as we've seen, the change in volume due to a change in pressure depends on the nature of the compression. For an isothermal compression

$$P = \frac{NkT}{V} \tag{0.2}$$

$$\Delta P = -NkT\frac{\Delta V}{V^2} \tag{0.3}$$

$$= -\frac{\Delta V}{V}P \tag{0.4}$$

$$B_{iso} = P \tag{0.5}$$

For an adiabatic compression

$$P = AV^{-\gamma} \tag{0.6}$$

$$\Delta P = -\gamma A\frac{\Delta V}{V^{\gamma+1}} \tag{0.7}$$

$$= -\gamma\frac{\Delta V}{V}P \tag{0.8}$$

$$B_{ad} = \gamma P \tag{0.9}$$

The speed of sound can be calculated from the bulk modulus and the material's mass density ρ as

$$c_s = \sqrt{\frac{B}{\rho}} \tag{0.10}$$

[For now, just take this as a god-given result.] Since a sound wave is a pressure wave that propagates through the medium at a high speed, pressure changes occur quickly so the adiabatic bulk modulus is the proper one to use in calculating c_s. [Note, however, that in deriving 0.6, we assumed the process was quasistatic, which means that the pressure has time to equalize throughout the medium. This is certainly not true for a sound wave (if it were, there would be no sound), so I'm not quite sure how this is reconciled with the formula above.]

Putting this worry aside, we can work out the speed of sound for an ideal gas. If the average molecular mass is m, then

$$\rho = \frac{Nm}{V} \tag{0.11}$$

so

$$c_s = \sqrt{\frac{\gamma PV}{Nm}} \tag{0.12}$$

$$= \sqrt{\frac{\gamma kT}{m}} \tag{0.13}$$

The root mean square speed of the molecules in an ideal gas is

$$v_{rms} = \sqrt{\frac{3kT}{m}} \tag{0.14}$$

As $\gamma = (f+2)/f$, $c_s < v_{rms}$.

The average molecular mass m can be found from the mass of a mole of dry air at room temperature and 1 atm pressure, which is 0.0289697 kg:

$$m = \frac{0.0289697}{6.02 \times 10^{23}} = 4.81 \times 10^{-26} \text{ kg} \tag{0.15}$$

Also,

$$\gamma = \frac{5+2}{5} = \frac{7}{5} \tag{0.16}$$

The speed of sound is

$$c_s = \sqrt{\frac{7 \times 1.38 \times 10^{-23} \times 293}{5 \times 4.81 \times 10^{-26}}} = 343 \text{ m s}^{-1} \tag{0.17}$$

which agrees with the measured value.

As the speed of sound depends only on the temperature, it does not depend on altitude above sea level, provided that the temperature remains constant. Although the air pressure decreases with altitude, if the temperature is the same, then the product $PV = NkT$ is also the same. However, it is true that, on average, sound travels faster in the summer than the winter due to the higher temperature. For example, on a frosty day when $T = 0^\circ$ C $= 273$ K, $c_s = 331$ m s^{-1}, while on a hot summer day with $T = 30^\circ$ C $= 303$ K, $c_s = 349$ m s^{-1}.

PINGBACKS

Pingback: Measuring heat capacity at constant volume