ATMOSPHERIC CONVECTION

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.40.

The barometric equation was derived under the assumption that the atmosphere is stable, so that at a given height z the pressure is equal to the weight of the column of air of unit cross-sectional area above that altitude. The barometric equation is

$$\frac{dP}{dz}=-\frac{mg}{kT}P \tag{1}$$

Earlier, we assumed that T was constant, which allowed us to integrate the equation. In reality, the temperature decreases with increasing z. If the temperature gradient $|dT/dz|$ passes a critical value, the density of the warmer air near the surface drops to a point where it starts to rise, resulting in convection, or the mass movement of air. Similarly, the density of the cooler air higher up is large enough that it falls towards the surface, so that a cycle is set up.

If we assume that the velocity of these air masses is high enough that little heat is lost or gained as the air moves vertically, we can use the adiabatic formula to analyze the situation. For adiabatic expansion, the pressure, volume and temperature are related by

$$VT^{f/2} = K \tag{2}$$

$$V^{\gamma}P = A \tag{3}$$

where K and A are constants and $\gamma=(f+2)/f$. Taking differentials we get

$$T^{f/2}dV+\frac{f}{2}VT^{\frac{f}{2}-1}dT = 0 \tag{4}$$

$$\gamma V^{\gamma-1}PdV+V^{\gamma}dP = 0 \tag{5}$$

Simplifying, we get

$$\begin{aligned}
TdV + \frac{f}{2} VdT &= 0 && (6)\\
\gamma PdV + VdP &= 0 && (7)\\
dT &= -\frac{2T}{fV} dV && (8)\\
dP &= -\frac{\gamma P}{V} dV && (9)\\
\frac{dT}{dP} &= \frac{2T}{f\gamma P} && (10)\\
&= \frac{2}{f+2} \frac{T}{P} && (11)
\end{aligned}$$

Now if $\frac{dT}{dz}$ is at the critical point, then 1 still applies and, since adiabatic expansion is just about to start, 11 should apply as well. From 1 we get

$$\begin{aligned}
\frac{dP}{P} &= -\frac{mg}{kT} dz && (12)\\
dT &= \frac{2T}{f+2} \frac{dP}{P} && (13)\\
\frac{dT}{dz} &= -\frac{2mg}{(f+2)k} && (14)
\end{aligned}$$

The average molecular mass m can be found from the mass of a mole of dry air at room temperature and 1 atm pressure, which is 0.0289697 kg:

$$m = \frac{0.0289697}{6.02 \times 10^{23}} = 4.81 \times 10^{-26} \text{ kg} \tag{15}$$

so the critical temperature gradient is

$$\frac{dT}{dz} = -\frac{2 \times 4.81 \times 10^{-26} \times 9.8}{(5+2)\,1.38 \times 10^{-23}} = -0.00976 \text{ deg m}^{-1} = -9.76 \text{ deg km}^{-1} \tag{16}$$

If the temperature gradient reaches around 10 degrees per kilometre, we can expect convection to occur. This is known as the *dry adiabatic lapse rate*.

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