THERMAL CONDUCTIVITY; R VALUES

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.56-1.57.

We can derive heuristically the Fourier heat conduction law as follows. Suppose we have a flat slab of area A and thickness Δx of some substance with the temperature on one side held at T_1 and on the other side at T_2, with $T_2 > T_1$. At what rate does heat Q flow through the slab?

If we consider an analogous situation with a ball rolling downhill, the rate at which the ball moves (its velocity) depends on the gradient of the slope. In the case of heat flow, we might therefore expect that the rate of heat flow depends on the temperature gradient across the slab, that is on $\Delta T/\Delta x = (T_2 - T_1)/\Delta x$. If the temperatures are constant over the area A, then we'd also expect the heat flow to be proportional to A, so if Q is the amount of heat that crosses the slab in time interval Δt, we get

$$\frac{Q}{\Delta t} = -k_t A \frac{\Delta T}{\Delta x} \tag{1}$$

where k_t is a constant called the *thermal conductivity*, which depends on the material of which the slab is made. The minus sign means that heat flows *down* the temperature gradient, so that if $T_2 > T_1$, then heat flows from T_2 to T_1. [In more complex situations, k_t could also depend on the temperature and other things, but we'll ignore that for now.] In principle, it is possible to derive k_t from the atomic nature of the material, but for now we'll assume that it's just a property of a material that must be measured. The units of k_t are therefore $[\text{watts}]\,\text{m}^{-1}\text{K}^{-1} = \text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$.

Example 1. $k_t = 0.026$ for air. For a layer of still air with a surface area of 1 m^2, a thickness of 1 mm and $\Delta T = 20$ K, we get

$$\frac{Q}{\Delta t} = 0.026 \times 1 \times \frac{20}{0.001} \tag{2}$$

$$= 520 \text{ watts} \tag{3}$$

Example 2. In the building trade, the thermal conductivity is often given in terms of the R value, which is defined as

$$R \equiv \frac{\Delta x}{k_t} \tag{4}$$

Since R depends on the inverse of the thermal conductivity and directly on the thickness of the insulating material, a larger R means a better insulator. For the 1 mm layer of still air in the previous example, we have

$$R_{air} = \frac{0.001}{0.026} = 0.0385\ \mathrm{m^2K\ s\ J^{-1}} \tag{5}$$

Given that k_t for glass 0.8, the R value of a 3.2 mm thick sheet of glass (a typical window) is

$$R_{glass} = \frac{0.0032}{0.8} = 0.004\ \mathrm{m^2K\ s\ J^{-1}} \tag{6}$$

Thus if there is a 1 mm layer of still air next to a window, it actually provides more insulation than the window glass itself.

For some reason, Schroeder makes us use the convoluted so-called 'English' units for R (even though we in the UK now use the metric system for most building quantities now). In the US, the units of R are $^{\circ}\mathrm{F\ ft^2 hr\ Btu^{-1}}$ where a British thermal unit (Btu) is defined as the energy required to raise one pound of water by 1°F. To convert to SI units of R, we need to convert 1 Btu to Joules. One Fahrenheit degree is $\frac{5}{9}$ of a Kelvin, there are 453.592 grams per pound, and 4.186 joules are required to raise 1 gram of water by 1 K. Therefore

$$1\ \mathrm{Btu} = (453.592)(4.186)\frac{5}{9} = 1054.85\ \mathrm{J} \tag{7}$$

One foot is 0.3048 m and 1 hour is 3600 seconds, so

$$1^{\circ}\mathrm{F\ ft^2 hr\ Btu^{-1}} = \frac{5(0.3048)^2(3600)}{9(1054.85)} = 0.176\ \mathrm{m^2K\ s\ J^{-1}} \tag{8}$$

The previous R values in English units are therefore

$$\begin{aligned} R_{air} &= 0.219^{\circ}\mathrm{F\ ft^2 hr\ Btu^{-1}} && (9)\\ R_{glass} &= 0.0227^{\circ}\mathrm{F\ ft^2 hr\ Btu^{-1}} && (10) \end{aligned}$$

The R values of a compound layer of two different materials is the sum of the individual R values. We can see this as follows. Suppose we have a compound layer composed of two materials: material 1 and material 2. The temperature on the exposed side of material 2 is T_2 and on the material 1 side is T_1. The temperature at the point where the two materials join is T_a. In the steady state, the rate of heat flow must be the same across the two layers (otherwise heat would build up somewhere) so from 1 (taking $A = 1$ for convenience; it drops out of the calculation anyway)

$$-\frac{Q}{\Delta t} = k_2\frac{T_2-T_a}{\Delta x_2} = k_1\frac{T_a-T_1}{\Delta x_1} \quad (11)$$

$$\frac{T_2-T_a}{R_2} = \frac{T_a-T_1}{R_1} \quad (12)$$

Now the overall rate of heat flow across the compound layer must also be the same. If we define R_c to be the effective R value of the compound layer, then

$$\frac{T_2-T_a}{R_2} = \frac{T_a-T_1}{R_1} = \frac{T_2-T_1}{R_c} \quad (13)$$

We thus have 2 equations in 2 unknowns (T_a and R). From the first equation, we can solve for T_a:

$$T_a = \frac{T_2R_1+T_1R_2}{R_1+R_2} \quad (14)$$

Substituting into the second equation we can solve for R_c:

$$\begin{aligned} R_c &= R_1\frac{T_2-T_1}{T_a-T_1} && (15)\\ &= (R_1+R_2)R_1\frac{T_2-T_1}{T_2R_1+T_1R_2-T_1(R_1+R_2)} && (16)\\ &= R_1+R_2 && (17) \end{aligned}$$

Example 3. Using this compound R value, we can estimate the rate of heat loss from a 1 m^2 single-pane window of thickness 3.2 mm, but with a 1 mm layer of still air on each side. The effective R value of this system is

$$R_c = 2\times R_{air}+R_{glass} = 0.081\ \mathrm{m^2K\ s\ J^{-1}} \quad (18)$$

When the temperature difference is $\Delta T = 20$ K, the rate of heat loss is

$$\frac{Q}{\Delta t} = \frac{A\Delta T}{R_c} = 247\text{ watts} \quad (19)$$

This compares with the heat loss through the glass on its own of $A\Delta T/R_{glass} =$ 5000 watts. Thus the air layer actually provides most of the insulation.

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