

## THERMAL CONDUCTIVITY OF HELIUM

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.64.

We can estimate the thermal conductivity of a gas such as helium using the approximate formula

$$(1) \quad k_t = \frac{C_V}{2V} \ell \bar{v}$$

where  $\bar{v}$  is the average molecular velocity, which we can approximate by the rms speed, which is

$$(2) \quad \bar{v} \approx v_{rms} = \sqrt{\frac{3kT}{m}}$$

The mean free path  $\ell$  is based on the idea that the mean path length is equal to the length of a cylinder of radius equal to the molecule's diameter and volume equal to the average volume per molecule  $V/N$ , so that

$$(3) \quad \ell = \frac{1}{4\pi r^2} \frac{V}{N}$$

where  $r$  is the radius of the molecule. The heat capacity is

$$(4) \quad C_V = \frac{1}{2} N f k = \frac{f}{2} \frac{PV}{T}$$

where  $f$  is the number of degrees of freedom of the molecule.

At room temperature and pressure we can work out  $k_t$  for helium by looking up a few properties using Google. The effective radius of a helium atom seems to depend a lot on which web page you find. It seems that the van der Waals radius is defined as half the distance between two nuclei when two non-bonded atoms are at their closest possible approach. The most commonly quoted value for helium is

$$(5) \quad r = 1.4 \times 10^{-10} \text{ m}$$

This gives a mean free path of

$$(6) \quad \ell = \frac{1}{4\pi r^2} \frac{V}{N} = \frac{1}{4\pi r^2} \frac{kT}{P} = 1.68 \times 10^{-7} \text{ m}$$

The mass of a helium-4 atom is about 4 atomic mass units or

$$(7) \quad m = 4 \times 1.66 \times 10^{-27} = 6.64 \times 10^{-27} \text{ kg}$$

The rms velocity at  $T = 300 \text{ K}$  is therefore

$$(8) \quad v_{rms} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{6.64 \times 10^{-27}}} = 1.37 \times 10^3 \text{ m s}^{-1}$$

Since helium is monatomic, it has only 3 degrees of freedom so  $f = 3$  and

$$(9) \quad \frac{C_V}{V} = \frac{3}{2} \frac{P}{T} = \frac{3}{2} \frac{10^5}{300} = 500 \text{ J m}^{-3} \text{K}^{-1}$$

Putting all this together gives an estimate of  $k_t$ :

$$(10) \quad k_t = \frac{1}{2} (500) (1.68 \times 10^{-7}) (1.37 \times 10^3) = 0.057 \text{ W m}^{-1} \text{K}^{-1}$$

This is only about half the measured value of around 0.142. Using a radius of around  $0.95 \times 10^{-10} \text{ m}$  gives a better result, and this value is given on a few web sites so who knows? In any case, we'd expect  $k_t$  for helium to be higher than air, since the lower mass of the molecule (it being a single atom) gives it a higher speed so it will transport energy faster.