We can get a rough estimate of Avogadro’s number from the thermal conductivity and other macroscopic quantities measurable for an ideal gas. Schroeder’s derivation of the thermal conductivity formula gives the result

\[ k_t = \frac{C_V}{2V} \ell \bar{v} \]  

(1)

Suppose we set up the system so that we have a box of volume \( V \) and cross-sectional area \( A \). We know that the average speed \( \bar{v} \) is approximately the rms speed, which is

\[ \bar{v} \approx v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3NkT}{Nm}} = \sqrt{\frac{3PV}{M}} \]  

(2)

where \( M \) is the total mass of the gas. If we measure the thermal conductivity and heat capacity, we can then get the mean free path:

\[ \ell = \frac{2V k_t}{C_V \bar{v}} = \frac{2k_t}{C_V} \sqrt{\frac{MV}{3P}} \]  

(3)

Schroeder’s expression for \( \ell \) is based on the idea that the mean path length is equal to the length of a cylinder of radius equal to the molecule’s diameter and volume equal to the average volume per molecule \( V/N \), so that

\[ \ell = \frac{1}{4\pi r^2} \frac{V}{N} \]  

(4)

where \( r \) is the radius of the molecule. To get \( N \) from this formula, we need to know \( r \), but this is a microscopic quantity which we’re assuming we don’t know. I can’t see any way of progressing from here unless we take a different value for \( \ell \). Since we’re after only a rough approximation of Avogadro’s number, we can take \( \ell \) to be the average distance between molecules, rather than the average distance between collisions. That is

\[ \ell \approx \left( \frac{V}{N} \right)^{1/3} \]  

(5)
We can now combine this with 3 to get an estimate for the number of molecules $N$:

$$N \approx \left( \frac{C_V}{2k_t} \right)^3 \left( \frac{3P}{MV} \right)^{3/2} V = \frac{1}{\sqrt{V}} \left( \frac{C_V}{2k_t} \right)^3 \left( \frac{3P}{M} \right)^{3/2} \quad (6)$$

Assuming we know the gas constant $R$, we can get the number of moles from the ideal gas law

$$n = \frac{PV}{RT} \quad (7)$$

so Avogadro’s number is roughly

$$N_A = \frac{N}{n} \approx RT\sqrt{P} \left( \frac{C_V}{2k_t} \right)^3 \left( \frac{3}{MV} \right)^{3/2} \quad (8)$$

As a check on this formula, we can verify that it’s dimensionless. $RT$ has the dimensions of $PV$, so dimensionally, the quantity is equivalent to

$$\left( \frac{C_V}{k_t} \right)^3 \left( \frac{P}{M} \right)^{3/2} \frac{1}{\sqrt{V}} \rightarrow \left[ \frac{\text{energy/K}}{\text{energy/s m K}} \right]^3 \left[ \frac{\text{force/m}^2}{\text{kg}} \right]^{3/2} \frac{1}{\text{m}^{3/2}} \quad (9)$$

$$= \left[ \text{s}^3 \text{m}^3 \right] \left[ \text{m}^{-1} \text{s}^{-2} \right]^{3/2} \text{m}^{-3/2} \quad (10)$$

$$= 1 \quad (11)$$

Thus the dimensions check out.

I don’t know if this is the solution Schroeder had in mind, or whether it’s possible to get $N_A$ using the formula 4 for the mean free path. If we’re not allowed to know a priori any microscopic quantities, that means we don’t know $k$ (Boltzmann’s constant), $m$ (the mass of a single molecule) or $r$ (the radius of a molecule). Together with $N$, that makes 4 unknowns, so we need 4 independent equations to find them. Comments welcome.

**Remark 1.** Thanks for the quick reply. I think what he’s saying is that we can take $\frac{4}{3} \pi r^3$ to be the volume per molecule in the liquid phase, so that

$$\frac{4}{3} \pi r^3 = \frac{V_{liq}}{N}$$

If we know that the volume in the gas phase is 1000 times that in the liquid, then

$$\ell \approx \frac{r V_{gas}}{3} \frac{N}{V_{liq}} = \frac{1000}{3} r$$
Hence $\ell$ is about 300 - 400 times larger than $r$. It’s interesting that he doesn’t actually work out the numbers, though (but of course neither did I), so it’s hard to say how good an estimate of $N_A$ this is.