VISCOSITY OF AN IDEAL GAS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 1.66.

We can treat viscosity in gases in a similar way to that used for thermal conductivity. A common situation involving viscosity is that of two horizontal, parallel flat plates with a gas or liquid sandwiched between them. If one plate moves parallel to the other, the gas between the plates exerts a drag force inhibiting the motion of the plates.

In the reference frame with the lower plate at rest and the upper plate moving at some speed u_x to the right, we'd expect the fluid between the plates to be moving at a speed that increases from zero next to the lower plate up to u_x next to the upper plate. This gradient in speed is the result of momentum transfer between adjacent layers in the fluid. Because of Newton's law of equal action and reaction, the horizontal drag force exerted on each plate is equal and opposite to the force on the fluid layer directly adjacent to the plate.

The guesstimate derivation given by Schroeder assumes that the force on each plate is proportional to the area A of the plate and to the relative speed of the upper and lower plates $u_{x,top} - u_{x,bottom}$, and inversely proportional to the distance Δz between the plates. The last two assumptions are equivalent to saying that the force is proportional to the velocity gradient du_x/dz. That is

$$\frac{F_x}{A} = \eta \frac{du_x}{dz} \tag{1}$$

where η is the *coefficient of viscosity* or just the *viscosity*. From its definition, it has units of [force] [area]$^{-1}$ [time] or N m^{-2}s.

By following a similar argument to that for thermal conductivity, we can get an estimate for η in the case of an ideal gas. We'll assume that the mean free path and average molecular velocity for the gas are the same as before:

$$\ell \approx \frac{1}{4\pi r^2}\frac{V}{N} \tag{2}$$

$$\bar{v} \approx \sqrt{\frac{3kT}{m}} \tag{3}$$

where r is the molecular radius and m the mass of one molecule. Then if we consider a thin horizontal slab of the gas between the plates, those molecules within a distance ℓ of the midpoint of the slab can cross the midpoint if they are travelling towards the midpoint. On average half the molecules in each half are travelling towards the midpoint so if the average horizontal momentum of the molecules on side i is p_i for $i = 1, 2$, then in a time Δt (that is, the time it takes an average molecule to travel ℓ) the momentum transferred is

$$\Delta p = \frac{1}{2}(p_1 - p_2) = \frac{M}{2}(u_{x,1} - u_{x,2}) = \frac{M}{2}\ell\frac{du_x}{dz} \tag{4}$$

where M is the total mass of gas in a slab of area A and thickness ℓ.

The average force per unit area of the plates is then

$$\frac{F_x}{A} = \frac{\Delta p}{A\Delta t} = \frac{M}{2A\Delta t}\ell\frac{du_x}{dz} = \frac{M}{2A\ell\Delta t}\ell^2\frac{du_x}{dz} = \frac{\rho}{2}\ell\bar{v}\frac{du_x}{dz} \tag{5}$$

where $\rho = M/A\ell$ is the mass density of the gas and $\bar{v} = \ell/\Delta t$ is the average speed.

For an ideal gas $\rho = mN/V$ so combining this with the above expressions for ℓ and $\bar{v}$ we get

$$\eta = \frac{\sqrt{3mkT}}{4\pi r^2} \tag{6}$$

Thus the viscosity for an ideal gas is independent of pressure and depends only on the temperature.

For air, the density is around 1 kg m^{-3}, $\ell \approx 1.5 \times 10^{-7}$ m and $\bar{v} \approx 500$ m s^{-1} so

$$\eta \approx 3.75 \times 10^{-5}\text{N m}^{-2}\text{s} \tag{7}$$

This is about double the value of 19 μPa s (1 Pascal is 1 N m^{-2}) given in the book, but it's not bad for a rough estimate.

PINGBACKS

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