DIFFUSION: A COUPLE OF EXAMPLES

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 1.67 - 1.68.

The process of diffusion obeys Fick's law

$$J_x = -D\frac{dn}{dx} \tag{1}$$

where J_x is the number of molecules per unit area diffusing in the x direction per unit time, D is the diffusion constant and $n = N/V$ is the particle density.

We can use Fick's law to do a few estimates of how long various types of diffusion processes take.

Example 1. Schroeder gives the diffusion constant D for sucrose (table sugar) in water at room temperature as $D = 5\times10^{-10}\text{m}^2\text{s}^{-1}$. If we take $\Delta t = 60$ s, how far can we expect a molecule of sucrose to diffuse? We can estimate this by taking Δx to be the distance over which diffusion has taken place. The volume of this region is $V = A\Delta x$ where A is the cross sectional area of the container. If N is the total number of molecules within this region, then, approximately,

$$n \approx \frac{N}{A\Delta x} \tag{2}$$

The flux can be calculated by taking Δt to be the time taken for the volume $A\Delta x$ to acquire the N molecules, so we get, again approximately

$$J_x \approx \frac{N}{A\Delta t} \tag{3}$$

From Fick's law we get

$$\frac{N}{A\Delta t} \approx D\frac{N}{A(\Delta x)^2} \tag{4}$$

$$\Delta x \approx \sqrt{D\Delta t} \tag{5}$$

With the values given, we have

$$\Delta x \approx 1.7\times10^{-4}\text{ m} \tag{6}$$

or about a fifth of a millimetre.

Example 2. As an example of diffusion in air, suppose a bottle of perfume is opened at one end of a room 10 m long. Schroeder gives $D = 2\times10^{-5}\ \mathrm{m}^2\mathrm{s}^{-1}$ for CO molecules in air at room temperature and atmospheric pressure. A perfume molecule is probably larger than a CO molecule, so its diffusion constant would be smaller, say, $D = 10^{-5}$. Using the same approximate formula as in the previous example, the time it would take for the perfume to diffuse to the other end of the room is

$$\Delta t \approx \frac{(\Delta x)^2}{D} = 10^7\ \mathrm{s} \tag{7}$$

which is about a third of a year. Certainly most odours released at some point in a room (including those embarrassing to the emitter, at times) travel the length of the room considerably faster than that (often less than a minute), so other processes (typically convection) are responsible for spreading them.

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