EINSTEIN SOLID

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 2.5 - 2.7.

A simple model of a solid proposed by Einstein in 1907 is that it consists of a collection of N oscillators with quantized energy units. We can think of each oscillator as a quantum harmonic oscillator, and each energy unit as a quantum of size $\hbar\omega$, but the concept applies to any system with energy units that are all the same size. In general, a solid with N oscillators can have q energy units to distribute amongst them, so the number of possible microstates of such a system is the number of ways of distributing q balls into N bins. This is a standard problem in combinatorics, and the solution goes as follows.

We can represent the q balls by Xs and the N bins by $N-1$ vertical bars, where each bar serves to separate the contents of one bin from its neighbour. Thus if we have $N=3$ and $q=4$, the possible microstates are

$$||XXXX$$
$$|XXXX|$$
$$XXXX||$$
$$|X|XXX$$
$$|XX|XX$$
$$|XXX|X$$
$$X||XXX$$
$$XX||XX$$
$$XXX||X$$
$$X|XXX|$$
$$XX|XX|$$
$$XXX|X|$$
$$X|X|XX$$
$$X|XX|X$$
$$XX|X|X$$

In general, the number of microstates is the number of ways of choosing q (or $N-1$) objects from a total of $q+N-1$ objects, without regard to order, which is just the binomial coefficient $\binom{q+N-1}{q}$. For the example just given,

$$\binom{q+N-1}{q} = \binom{6}{4} = 15 \tag{1}$$

which corresponds to the 15 cases listed above.

In his problem 2.5, Schroeder asks us to list the microstates for several other values of N and q, but this gets pretty tedious and the general idea should be obvious from the above. We'll just list the number of microstates for each case.

N	q	$\binom{q+N-1}{q}$
3	5	21
3	6	28
4	2	10
4	3	20
1	anything	1
anything	1	N
30	30	59132290782430712

Admittedly, Schroeder does tell us not to attempt to list all the microstates for the last line(!)

Well OK, just one more example, with $N=4$ and $q=2$.

$$\begin{gathered}|||XX\\ ||XX|\\ |XX||\\ XX|||\\ ||X|X\\ |X|X|\\ X|X||\\ |X||X\\ X|||X\\ X||X|\end{gathered}$$

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