

## EINSTEIN SOLID

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 2.5 - 2.7.

A simple model of a solid proposed by Einstein in 1907 is that it consists of a collection of  $N$  oscillators with quantized energy units. We can think of each oscillator as a quantum harmonic oscillator, and each energy unit as a quantum of size  $\hbar\omega$ , but the concept applies to any system with energy units that are all the same size. In general, a solid with  $N$  oscillators can have  $q$  energy units to distribute amongst them, so the number of possible microstates of such a system is the number of ways of distributing  $q$  balls into  $N$  bins. This is a standard problem in combinatorics, and the solution goes as follows.

We can represent the  $q$  balls by Xs and the  $N$  bins by  $N - 1$  vertical bars, where each bar serves to separate the contents of one bin from its neighbour. Thus if we have  $N = 3$  and  $q = 4$ , the possible microstates are

||XXXX  
|XXXX|  
XXXX||  
|X|XXX  
|XX|XX  
|XXX|X  
X||XXX  
XX||XX  
XXX||X  
X|XXX|  
XX|XX|  
XXX|X|  
X|X|XX  
X|XX|X  
XX|X|X

In general, the number of microstates is the number of ways of choosing  $q$  (or  $N - 1$ ) objects from a total of  $q + N - 1$  objects, without regard to order, which is just the binomial coefficient  $\binom{q+N-1}{q}$ . For the example just given,

$$\binom{q+N-1}{q} = \binom{6}{4} = 15 \quad (1)$$

which corresponds to the 15 cases listed above.

In his problem 2.5, Schroeder asks us to list the microstates for several other values of  $N$  and  $q$ , but this gets pretty tedious and the general idea should be obvious from the above. We'll just list the number of microstates for each case.

$N$	$q$	$\binom{q+N-1}{q}$
3	5	21
3	6	28
4	2	10
4	3	20
1	anything	1
anything	1	$N$
30	30	59132290782430712

Admittedly, Schroeder does tell us not to attempt to list all the microstates for the last line(!)

Well OK, just one more example, with  $N = 4$  and  $q = 2$ .

|||XX  
 ||XX|  
 |XX||  
 XX||  
 ||X|X  
 |X|X|  
 X|X||  
 |X||X  
 X|||X  
 X||X|

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