INTERACTING EINSTEIN SOLIDS: A FEW EXAMPLES

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 2.9 - 2.10.

Here are a few more examples of the probabilities of various macrostates in two interacting Einstein solids. As before, we have two solids, A and B, containing N_A and N_B oscillators and q_A and q_B quanta of energy, with $q_A + q_B = q =$ constant. For any particular partition of the quanta, that is, for particular values of q_A and q_B, the total number of microstates available to the compound system is

$$\Omega_{total} = \Omega_A \Omega_B = \binom{q_A + N_A - 1}{q_A}\binom{q_B + N_B - 1}{q_B} \tag{1}$$

The overall number of microstates is

$$\Omega_{overall} = \binom{q + N_A + N_B - 1}{q} \tag{2}$$

Example 1. Consider a simple system with $N_A = N_B = 3$ and $q = 6$ as shown in Fig 2.4 in Schroeder. Using Maple to calculate the binomial coefficients (Maple has a 'binomial' function that does this automatically) and produce the plot, we have

$$\Omega_{overall} = \binom{6+3+3-1}{6} = 462 \tag{3}$$

$$\Omega_A = \binom{q_A + 2}{q_A} \tag{4}$$

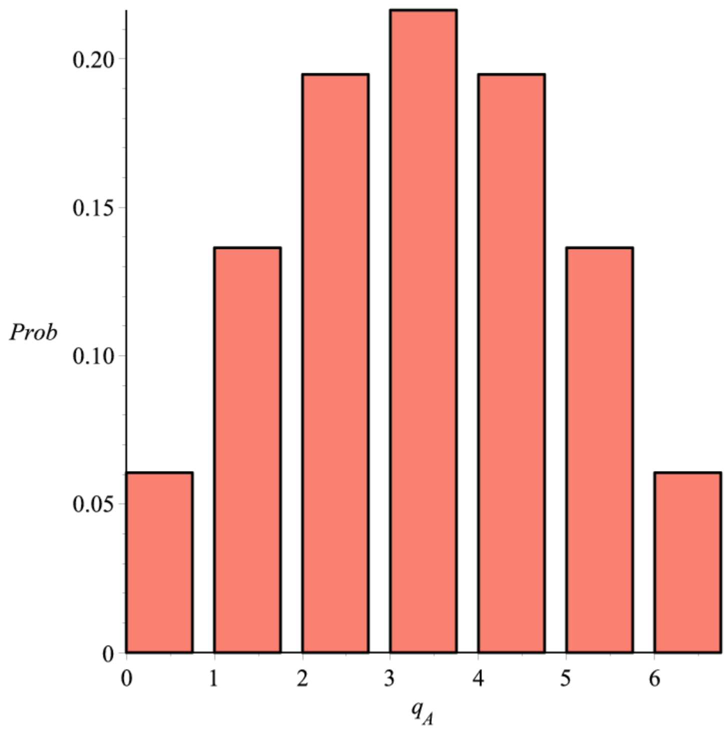
$$\Omega_B = \binom{q - q_A + 2}{q - q_A} = \binom{8 - q_A}{6 - q_A} \tag{5}$$

$$Prob(q_A) = \frac{\Omega_A \Omega_B}{\Omega_{overall}} \tag{6}$$

Plugging in the numbers, we get

q_A	Ω_A	Ω_B	Ω_{total}	$Prob(q_A)$
0	1	28	28	0.061
1	3	21	63	0.136
2	6	15	90	0.195
3	10	10	100	0.216
4	15	6	90	0.195
5	21	3	63	0.136
6	28	1	28	0.061

A bar chart of the probabilities looks like this:



As before, the most likely state is when the energy is equally distributed between the two solids.

Example 2. Now we'll see what happens if one solid has more oscillators to store energy than the other one. We'll take $N_A = 6$, $N_B = 4$ and $q = 6$. We now have

$$\Omega_{overall} = \binom{6+6+4-1}{6} = 5005 \tag{7}$$

$$\Omega_A = \binom{q_A+5}{q_A} \tag{8}$$

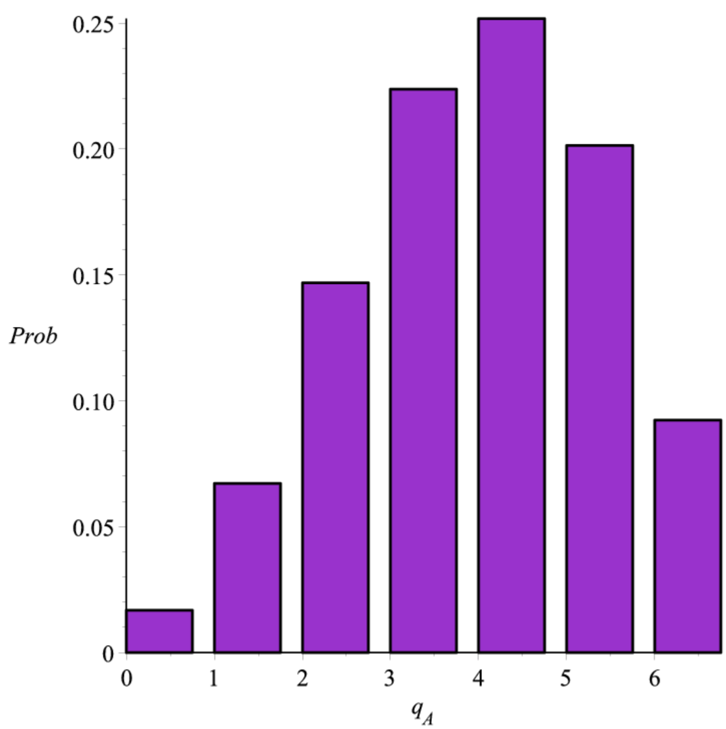
$$\Omega_B = \binom{q-q_A+3}{q-q_A} = \binom{9-q_A}{6-q_A} \tag{9}$$

$$Prob(q_A) = \frac{\Omega_A\Omega_B}{\Omega_{overall}} \tag{10}$$

Plugging in the numbers, we get

q_A	Ω_A	Ω_B	Ω_{total}	$Prob(q_A)$
0	1	84	84	0.017
1	6	56	336	0.067
2	21	35	735	0.147
3	56	20	1120	0.224
4	126	10	1260	0.252
5	252	4	1008	0.201
6	462	1	462	0.092

A bar chart of the probabilities looks like this:



Since solid A has more oscillators, the probabilities are skewed towards more of the quanta being stored in solid A than in solid B.

Example 3. Now we'll ramp things up a bit and consider two solids with $N_A = 200$, $N_B = 100$ and $q = 100$. As there are 101 macrostates, we won't list all the various states. The formulas in this case are

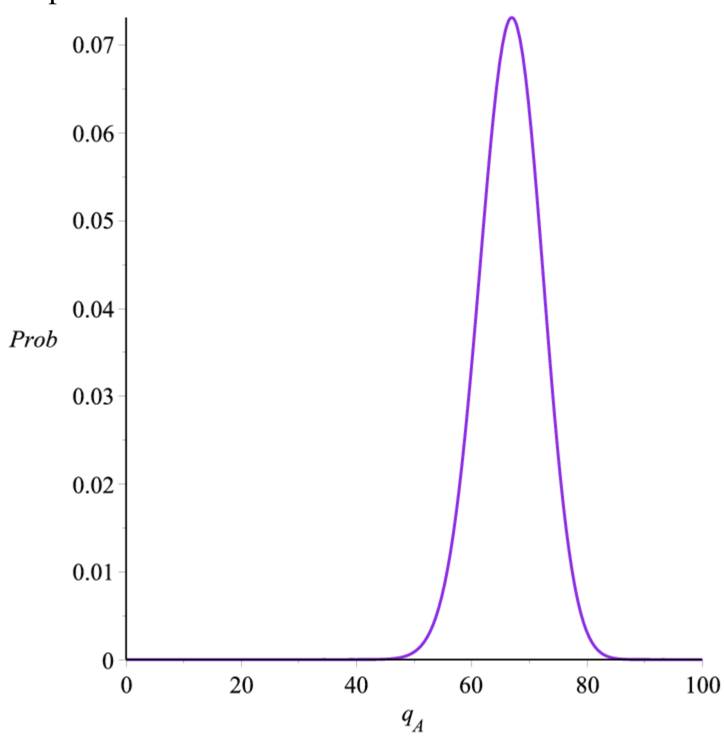
$$\Omega_{overall} = \binom{200+100+100-1}{100} = 1.681 \times 10^{96} \tag{11}$$

$$\Omega_A = \binom{q_A+199}{q_A} \tag{12}$$

$$\Omega_B = \binom{q-q_A+99}{q-q_A} = \binom{199-q_A}{100-q_A} \tag{13}$$

$$Prob(q_A) = \frac{\Omega_A \Omega_B}{\Omega_{overall}} \tag{14}$$

The plot is



The maximum probability of 0.073 occurs at $q_A = 67$ and the minimum of 2.69×10^{-38} at $q_A = 0$. As solid A contains $\frac{2}{3}$ of the oscillators, the maximum probability is when $\frac{2}{3}$ of the energy is stored in solid A. Notice how vanishing small is the chance of finding the system in a macrostate with anything less than about $q_A = 45$. Thus even though all microstates are equally probable, it is overwhelmingly likely that the energy will be more or less evenly distributed over all the oscillators.

PINGBACKS

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