

## BLACK HOLE ENTROPY

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 2.42.

In our study of general relativity, we've seen a formula for the entropy of a black hole. Schroeder takes a different approach, outlined in his problem 2.42.

First, we can calculate the radius of a black hole using Newtonian physics (which actually turns out to be the same as that calculated from general relativity). We take the radius  $r$  of a black hole of mass  $M$  to be such that the escape velocity at the surface is equal to the speed of light  $c$ . That is

$$\frac{GM}{r^2} = \frac{c^2}{r} \quad (1)$$

$$r = \frac{GM}{c^2} \quad (2)$$

If we assume that the entropy of a black hole is  $Nk$  multiplied by some logarithm, we can use the argument given earlier to say that an order of magnitude estimate of the entropy is

$$S \sim Nk \quad (3)$$

Therefore, we need an estimate of the number of particles that went into constructing the black hole. The argument goes as follows. Since there is no way to tell what kind of mass or energy went into the construction of the black hole (assuming no charge or angular momentum are involved), we'd like to take the maximum number of particles that could be used. In other words, we'd like to find the lowest energy  $mc^2$  of a massive particle, or  $h\nu$  for a photon, that can be used. Long wavelength photons have the lowest energy, so if we take the lowest possible energy to be the photon with a wavelength equal to the radius of the black hole, we have

$$\lambda = r = \frac{GM}{c^2} \quad (4)$$

$$E = h\nu = \frac{hc}{\lambda} = \frac{hc^3}{GM} \quad (5)$$

The total energy of the black hole is  $Mc^2$  so the number of such photons is

$$N = \frac{Mc^2}{E} = \frac{GM^2}{hc} \quad (6)$$

An estimate of the entropy is therefore

$$S \sim \frac{GM^2}{hc} k \quad (7)$$

Apart from the factor of  $8\pi^2$ , this agrees with the actual value:

$$S = \frac{8\pi^2 GM^2}{hc} k \quad (8)$$

For a solar mass black hole

$$r = \frac{(6.67 \times 10^{-11}) (2 \times 10^{30})}{(3 \times 10^8)^2} = 1482 \text{ m} \quad (9)$$

$$S = \frac{8\pi^2 (6.67 \times 10^{-11}) (2 \times 10^{30})^2}{(6.62 \times 10^{-34}) (3 \times 10^8)} (1.38 \times 10^{-23}) = 1.46 \times 10^{54} \text{ J K}^{-1} \quad (10)$$

This is vastly greater than our order of magnitude estimate for the entropy of the sun, which is around  $1.66 \times 10^{34} \text{ J K}^{-1}$ .

#### PINGBACKS

Pingback: Temperature of a black hole