TEMPERATURE OF AN EINSTEIN SOLID

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.5.

The definition of temperature in terms of entropy is

$$\frac{1}{T}\equiv\frac{\partial S}{\partial U} \tag{1}$$

Given a formula for the entropy of a system, we can use this relation to work out its temperature. As an example, we'll look at the Einstein solid in the low temperature case where the number q of energy quanta (each of size ε) is much less than the number N of oscillators: $q\ll N$. The multiplicity of such a system is approximately

$$\Omega\approx\left(\frac{Ne}{q}\right)^{q} \tag{2}$$

The total energy of the system is $U=q\varepsilon$ so we can write this in terms of U as

$$\Omega\approx\left(\frac{N\varepsilon e}{U}\right)^{q} \tag{3}$$

so the entropy is

$$\begin{aligned} S &= k\ln\Omega && (4)\\ &= \frac{kU}{\varepsilon}\left(\ln\left(\varepsilon N\right)+1-\ln U\right) && (5)\end{aligned}$$

The partial derivative in 1 implies that we're holding N fixed, so we get, using the product rule:

$$\frac{1}{T} = \frac{k}{\varepsilon}\left(\ln\left(\varepsilon N\right)+1-\ln U\right)-\frac{k}{\varepsilon} \tag{6}$$

$$= \frac{k}{\varepsilon}\left(\ln\left(\varepsilon N\right)-\ln U\right) \tag{7}$$

$$\ln U = \ln\left(\varepsilon N\right)-\frac{\varepsilon}{kT} \tag{8}$$

$$U = N\varepsilon e^{-\varepsilon/kT} \tag{9}$$

Since we assumed $q \ll N$, this is equivalent to requiring $U = q\varepsilon \ll N\varepsilon$, so this result is valid only for low temperatures, as we'd expect. [Note that for high temperatures, $e^{-\varepsilon/kT} \to 1$ so $U \to N\varepsilon$ which violates the assumption $U \ll N\varepsilon$.]

Schroeder works out the energy-temperature relation for the other extreme $U \gg N\varepsilon$ in his section 3.1, with the result

$$U = NkT \tag{10}$$

In this case, there are enough energy quanta that every degree of freedom in all oscillators is excited and since there are two degrees of freedom per oscillator, this agrees with the equipartition theorem which says that every degree of freedom has an associated $\frac{1}{2}kT$ of kinetic energy.

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