TEMPERATURE OF A BLACK HOLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.7.

The entropy of a black hole is

$$S=\frac{8\pi^2GM^2}{hc}k \tag{0.1}$$

Taking $U=Mc^2$ as the energy of a black hole, we can write this as

$$S=\frac{8\pi^2Gk}{hc^5}U^2 \tag{0.2}$$

The temperature is therefore

$$T=\left(\frac{\partial S}{\partial U}\right)^{-1}=\frac{hc^5}{16\pi^2GkU}=\frac{hc^3}{16\pi^2GkM} \tag{0.3}$$

which agrees with our earlier result from general relativity.

For a solar mass black hole, this gives a value of

$$\begin{aligned} T &= \frac{\left(6.62\times10^{-34}\right)\left(3\times10^{8}\right)^3}{16\pi^2\left(6.67\times10^{-11}\right)\left(1.38\times10^{-23}\right)\left(2\times10^{30}\right)} && (0.4)\\ &= 6\times10^{-8}\text{ K} && (0.5)\end{aligned}$$

Not only are black holes dark, they are also very cold.

The entropy-versus-energy curve 0.2 is a parabola so its slope $\frac{\partial S}{\partial U}$ increases as U increases. As we've seen, this means that a black hole has negative heat capacity, and thus decreases in temperature as more energy (mass) is added. This is also clear from 0.3, since $T\propto\frac{1}{M}$.