

TEMPERATURE OF A BLACK HOLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.7.

The entropy of a black hole is

$$S = \frac{8\pi^2 GM^2}{hc} k \quad (1)$$

Taking $U = Mc^2$ as the energy of a black hole, we can write this as

$$S = \frac{8\pi^2 Gk}{hc^5} U^2 \quad (2)$$

The temperature is therefore

$$T = \left(\frac{\partial S}{\partial U} \right)^{-1} = \frac{hc^5}{16\pi^2 GkU} = \frac{hc^3}{16\pi^2 GkM} \quad (3)$$

which agrees with our earlier result from general relativity.

For a solar mass black hole, this gives a value of

$$T = \frac{(6.62 \times 10^{-34}) (3 \times 10^8)^3}{16\pi^2 (6.67 \times 10^{-11}) (1.38 \times 10^{-23}) (2 \times 10^{30})} \quad (4)$$

$$= 6 \times 10^{-8} \text{ K} \quad (5)$$

Not only are black holes dark, they are also very cold.

The entropy-versus-energy curve 2 is a parabola so its slope $\frac{\partial S}{\partial U}$ increases as U increases. As we've seen, this means that a black hole has negative heat capacity, and thus decreases in temperature as more energy (mass) is added. This is also clear from 3, since $T \propto \frac{1}{M}$.