TEMPERATURE OF A BLACK HOLE

The entropy of a black hole is

$$S = \frac{8\pi^2 G M^2}{hc} k$$

(1)

Taking $U = Mc^2$ as the energy of a black hole, we can write this as

$$S = \frac{8\pi^2 G k}{hc^5} U^2$$

(2)

The temperature is therefore

$$T = \left( \frac{\partial S}{\partial U} \right)^{-1} = \frac{hc^5}{16\pi^2 Gk U} = \frac{hc^3}{16\pi^2 Gk M}$$

(3)

which agrees with our earlier result from general relativity.

For a solar mass black hole, this gives a value of

$$T = \frac{(6.62 \times 10^{-34}) (3 \times 10^8)^3}{16\pi^2 (6.67 \times 10^{-11})(1.38 \times 10^{-23}) (2 \times 10^{30})}$$

(4)

$$= 6 \times 10^{-8} \text{ K}$$

(5)

Not only are black holes dark, they are also very cold.

The entropy-versus-energy curve is a parabola so its slope $\frac{\partial S}{\partial M}$ increases as $U$ increases. As we've seen, this means that a black hole has negative heat capacity, and thus decreases in temperature as more energy (mass) is added. This is also clear from since $T \propto \frac{1}{M}$. 