

## ENTROPY OF A STAR

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.15.

The heat capacity of a star was estimated using the virial theorem as

$$(1) \quad C_V = -\frac{3}{2}Nk$$

where  $N$  is the number of particles (typically dissociated protons and electrons) in the star. A negative heat capacity is typical of gravitationally bound systems.

We can use this to work out the entropy from the formula

$$(2) \quad S = \int \frac{C_V(T)}{T} dT$$

$$(3) \quad = -\frac{3}{2}Nk \ln T + f(N, V)$$

where  $f$  is some function that depends on  $N$  and the volume  $V$ , but not on  $T$ .

The total energy of a gravitationally bound system is negative and, from the virial theorem, we have

$$(4) \quad U = -K = -\frac{3}{2}NkT$$

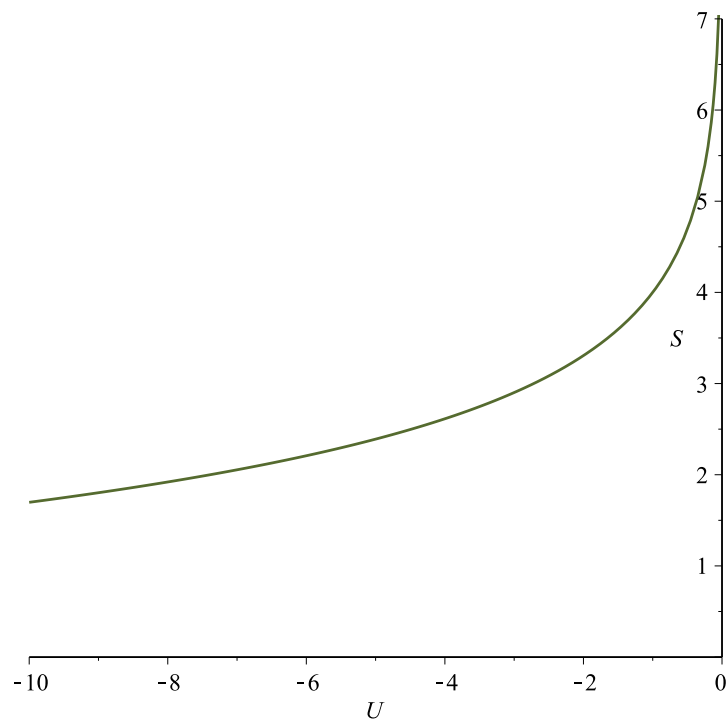
where  $K$  is the kinetic energy and the formula is obtained from the equipartition theorem. Thus the entropy can be written in terms of the energy as

$$(5) \quad S = -\frac{3}{2}Nk \ln \left| \frac{2U}{3Nk} \right| + f(N, V)$$

We can incorporate everything inside the logarithm except for  $U$  into the function  $f$  (call the new function  $g$ , say), so that

$$(6) \quad S = -\frac{3}{2}Nk \ln |U| + g(N, V)$$

The general shape of this curve is like this (units on the axes are arbitrary as I'm just trying to show the shape of the graph):



The graph is concave upwards, which is typical of systems with negative heat capacity as we discussed earlier. For sufficiently low (large negative) values of  $U$ , the graph would go negative, but I would guess that the temperature at that point would be higher than anything found in real stars so that would never happen. Note that as  $U \rightarrow 0$ ,  $S$  effectively becomes infinite. At  $U = 0$ , however, the system becomes gravitationally unbound, so the particles would presumably then be able to wander over the entire universe, giving them an infinite number of possible states.