ENTROPY OF A STAR

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.15.

The heat capacity of a star was estimated using the virial theorem as

$$C_V = -\frac{3}{2}Nk \tag{1}$$

where N is the number of particles (typically dissociated protons and electrons) in the star. A negative heat capacity is typical of gravitationally bound systems.

We can use this to work out the entropy from the formula

$$\begin{aligned} S &= \int \frac{C_V(T)}{T}dT \qquad (2) \\ &= -\frac{3}{2}Nk\ln T + f(N,V) \qquad (3) \end{aligned}$$

where f is some function that depends on N and the volume V, but not on T.

The total energy of a gravitationally bound system is negative and, from the virial theorem, we have

$$U = -K = -\frac{3}{2}NkT \tag{4}$$

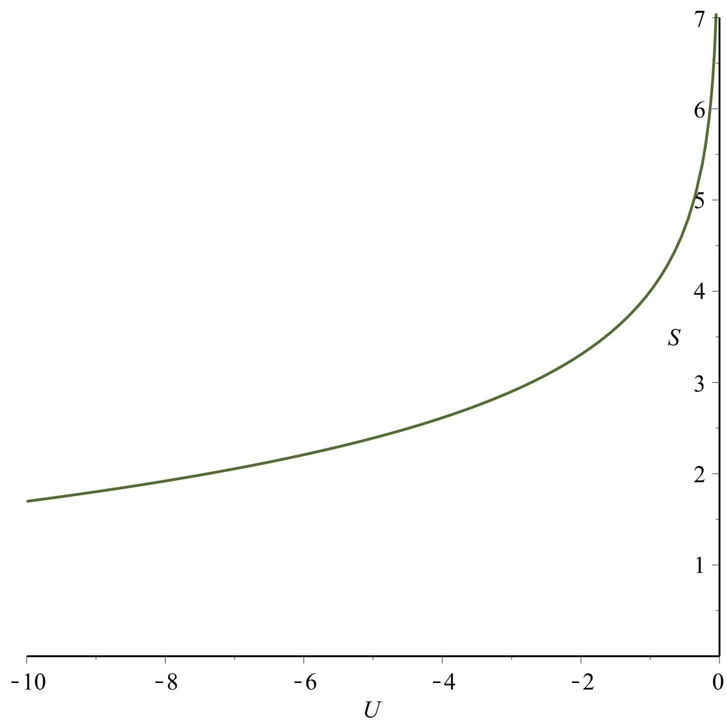
where K is the kinetic energy and the formula is obtained from the equipartition theorem. Thus the entropy can be written in terms of the energy as

$$S = -\frac{3}{2}Nk\ln\left|\frac{2U}{3Nk}\right| + f(N,V) \tag{5}$$

We can incorporate everything inside the logarithm except for U into the function f (call the new function g, say), so that

$$S = -\frac{3}{2}Nk\ln|U| + g(N,V) \tag{6}$$

The general shape of this curve is like this (units on the axes are arbitrary as I'm just trying to show the shape of the graph):



The graph is concave upwards, which is typical of systems with negative heat capacity as we discussed earlier. For sufficiently low (large negative) values of U, the graph would go negative, but I would guess that the temperature at that point would be higher than anything found in real stars so that would never happen. Note that as $U \to 0$, S effectively becomes infinite. At $U = 0$, however, the system becomes gravitationally unbound, so the particles would presumably then be able to wander over the entire universe, giving them an infinite number of possible states.