# ENTROPY OF ADIABATIC COMPRESSION

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.32.

The correspondence between the thermodynamic identity for an infinitesimal process

$$dU = TdS - PdV \tag{1}$$

and the heat plus work relation for the change in internal energy:

$$dU = Q + W \tag{2}$$

in which we equate $Q = TdS$ and $W = -PdV$ is valid only for quasistatic processes. As an example of a non-quasistatic process, suppose we have a cylinder containing one litre of air at room temperature (300 K) and atmospheric pressure ($10^5$ N m$^{-2}$), with a massless piston affixed to one end. If the piston is hit very hard so that it compresses the air very quickly (so that the piston moves faster than the speed of sound in air, say), then the quasistatic assumption breaks down because the pressure within the cylinder doesn't have a chance to equalize as the piston is being pushed in. This is an example of extreme adiabatic compression, in that the compression occurs so quickly that there is no heat gained or lost by the gas.

If the piston is compressed with a force of 2000 N over a distance of 1 mm, and the piston's surface area is 0.01 m$^2$, then the work done on the gas is the force times the distance, or

$$W = (2000)\left(10^{-3}\right) = 2 \text{ J} \tag{3}$$

As the volume of the cylinder changes by only $dV = -\left(10^{-3}\right)\left(10^{-2}\right) = -10^{-5}$ m$^3$, however, the $-PdV$ term is only

$$-PdV = 10^5 \times 10^{-5} = 1 \text{ J} \tag{4}$$

As the process is adiabatic, $Q = 0$ as no heat is exchanged, so the change in energy of the gas is

$$dU = W = 2 \text{ J} \tag{5}$$

The entropy change is (after allowing the shock wave generated by the sudden compression to dissipate):

$$dS = \frac{dU + PdV}{T} = \frac{1}{300} = 3.33 \times 10^{-3}\ \mathrm{J\ K}^{-1} \tag{6}$$

In this case, only half the work done on the gas goes into compressing it, with the remainder serving to heat up the gas (so technically this last equation should be an integral over temperature, but since we're dealing with very small changes, we can say that the temperature is approximately constant).