

THERMODYNAMIC PROPERTIES OF A 2-DIM IDEAL GAS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.39.

We now revisit the 2-d ideal gas for which the Sackur-Tetrode equation is

$$(1) \quad S = Nk \left[\ln \frac{2\pi mAU}{(hN)^2} + 2 \right]$$

where A is the area occupied by the gas, N is the number of molecules, each of mass m , and U is the total energy. We can work out the temperature, pressure and chemical potential by applying the thermodynamic identity adapted for 2 dimensions (by replacing the volume V by the area A):

$$(2) \quad dU = TdS - PdA + \mu dN$$

The temperature is determined from the entropy as

$$(3) \quad \frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_{A,N}$$

$$(4) \quad = Nk \frac{(hN)^2}{2\pi mAU} \frac{2\pi mA}{(hN)^2}$$

$$(5) \quad = \frac{Nk}{U}$$

This just gives us the formula from the equipartition theorem for a system with 2 degrees of freedom:

$$(6) \quad U = \frac{2}{2}NkT = NkT$$

The pressure can be obtained from

$$(7) \quad P = T \left(\frac{\partial S}{\partial A} \right)_{U,N}$$

$$(8) \quad = Nk \frac{(hN)^2}{2\pi mA U} \frac{2\pi m U}{(hN)^2}$$

$$(9) \quad = \frac{NkT}{A}$$

This is just the 2-dim analogue of the ideal gas law:

$$(10) \quad PA = NkT$$

Finally, chemical potential is defined in terms of the entropy as

$$(11) \quad \mu \equiv -T \left(\frac{\partial S}{\partial N} \right)_{U,A}$$

$$(12) \quad = -kT \left[\ln \frac{2\pi mA U}{(hN)^2} + 2 \right] - NkT \left(-\frac{2}{N} \right)$$

$$(13) \quad = -kT \ln \frac{2\pi mA U}{(hN)^2}$$

$$(14) \quad = -kT \ln \left(\frac{A}{N} \frac{2\pi mkT}{h^2} \right)$$

We can compare this to the chemical potential for a 3-d ideal gas

$$(15) \quad \mu = -kT \ln \left[\frac{V}{N} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} \right]$$

The only differences are the replacement of V by A and the change in the exponent inside the logarithm from $\frac{3}{2}$ to 1. The latter arises from the derivation of the multiplicity, where the exponent depends on the number of degrees of freedom in the system. For a 3-d gas, there are $3N$ degrees of freedom, while for a 2-d gas, there are $2N$. Thus the exponent in the 2-d case is $\frac{2}{3}$ that in the 3-d case. [You'd need to follow through the derivation in detail to see the difference, but basically that's where it comes from.]