

## THERMODYNAMIC PROPERTIES OF A 2-DIM IDEAL GAS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.39.

We now revisit the 2-d ideal gas for which the Sackur-Tetrode equation is

$$S = Nk \left[ \ln \frac{2\pi mAU}{(hN)^2} + 2 \right] \quad (1)$$

where  $A$  is the area occupied by the gas,  $N$  is the number of molecules, each of mass  $m$ , and  $U$  is the total energy. We can work out the temperature, pressure and chemical potential by applying the thermodynamic identity adapted for 2 dimensions (by replacing the volume  $V$  by the area  $A$ ):

$$dU = TdS - PdA + \mu dN \quad (2)$$

The temperature is determined from the entropy as

$$\frac{1}{T} = \left( \frac{\partial S}{\partial U} \right)_{A,N} \quad (3)$$

$$= Nk \frac{(hN)^2}{2\pi mAU} \frac{2\pi mA}{(hN)^2} \quad (4)$$

$$= \frac{Nk}{U} \quad (5)$$

This just gives us the formula from the equipartition theorem for a system with 2 degrees of freedom:

$$U = \frac{2}{2} NkT = NkT \quad (6)$$

The pressure can be obtained from

$$P = T \left( \frac{\partial S}{\partial A} \right)_{U,N} \quad (7)$$

$$= Nk \frac{(hN)^2}{2\pi mAU} \frac{2\pi mU}{(hN)^2} \quad (8)$$

$$= \frac{NkT}{A} \quad (9)$$

This is just the 2-dim analogue of the ideal gas law:

$$PA = NkT \quad (10)$$

Finally, chemical potential is defined in terms of the entropy as

$$\mu \equiv -T \left( \frac{\partial S}{\partial N} \right)_{U,A} \quad (11)$$

$$= -kT \left[ \ln \frac{2\pi mAU}{(hN)^2} + 2 \right] - NkT \left( -\frac{2}{N} \right) \quad (12)$$

$$= -kT \ln \frac{2\pi mAU}{(hN)^2} \quad (13)$$

$$= -kT \ln \left( \frac{A}{N} \frac{2\pi mkT}{h^2} \right) \quad (14)$$

We can compare this to the chemical potential for a 3-d ideal gas

$$\mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2\pi mkT}{h^2} \right)^{3/2} \right] \quad (15)$$

The only differences are the replacement of  $V$  by  $A$  and the change in the exponent inside the logarithm from  $\frac{3}{2}$  to 1. The latter arises from the derivation of the multiplicity, where the exponent depends on the number of degrees of freedom in the system. For a 3-d gas, there are  $3N$  degrees of freedom, while for a 2-d gas, there are  $2N$ . Thus the exponent in the 2-d case is  $\frac{2}{3}$  that in the 3-d case. [You'd need to follow through the derivation in detail to see the difference, but basically that's where it comes from.]