THERMODYNAMIC PROPERTIES OF A 2-DIM IDEAL GAS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 3.39.

We now revisit the 2-d ideal gas for which the Sackur-Tetrode equation is

$$S = Nk\left[\ln\frac{2\pi mAU}{(hN)^2} + 2\right] \tag{1}$$

where A is the area occupied by the gas, N is the number of molecules, each of mass m, and U is the total energy. We can work out the temperature, pressure and chemical potential by applying the thermodynamic identity adapted for 2 dimensions (by replacing the volume V by the area A):

$$dU = TdS - PdA + \mu dN \tag{2}$$

The temperature is determined from the entropy as

$$\begin{aligned}\frac{1}{T} &= \left(\frac{\partial S}{\partial U}\right)_{A,N} &(3)\\ &= Nk\frac{(hN)^2}{2\pi mAU}\frac{2\pi mA}{(hN)^2} &(4)\\ &= \frac{Nk}{U} &(5)\end{aligned}$$

This just gives us the formula from the equipartition theorem for a system with 2 degrees of freedom:

$$U = \frac{2}{2}NkT = NkT \tag{6}$$

The pressure can be obtained from

$$\begin{aligned} P &= T\left(\frac{\partial S}{\partial A}\right)_{U,N} && (7)\\ &= Nk\frac{(hN)^2}{2\pi mAU}\frac{2\pi mU}{(hN)^2} && (8)\\ &= \frac{NkT}{A} && (9)\end{aligned}$$

This is just the 2-dim analogue of the ideal gas law:

$$PA = NkT \tag{10}$$

Finally, chemical potential is defined in terms of the entropy as

$$\begin{aligned} \mu &\equiv -T\left(\frac{\partial S}{\partial N}\right)_{U,A} && (11)\\ &= -kT\left[\ln\frac{2\pi mAU}{(hN)^2}+2\right]-NkT\left(-\frac{2}{N}\right) && (12)\\ &= -kT\ln\frac{2\pi mAU}{(hN)^2} && (13)\\ &= -kT\ln\left(\frac{A}{N}\frac{2\pi mkT}{h^2}\right) && (14)\end{aligned}$$

We can compare this to the chemical potential for a 3-d ideal gas

$$\mu = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] \tag{15}$$

The only differences are the replacement of V by A and the change in the exponent inside the logarithm from $\frac{3}{2}$ to 1. The latter arises from the derivation of the multiplicity, where the exponent depends on the number of degrees of freedom in the system. For a 3-d gas, there are $3N$ degrees of freedom, while for a 2-d gas, there are $2N$. Thus the exponent in the 2-d case is $\frac{2}{3}$ that in the 3-d case. [You'd need to follow through the derivation in detail to see the difference, but basically that's where it comes from.]