HEAT ENGINES

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.1.

A heat engine is a system that absorbs heat and converts part of that heat into work, which can be extracted to perform other tasks. In practice, all heat engines absorb a certain amount of heat and can convert only part of that heat into work, with the remainder being expelled back into the environment. An idealized heat engine absorbs an amount of heat Q_h from a thermal reservoir at constant temperature T_h, converts an amount W of this heat into work, and expels the remainder Q_c into another thermal reservoir at a constant temperature T_c. The efficiency e of the heat engine is the fraction of Q_h that is converted into work, so

$$e=\frac{W}{Q_h}=\frac{Q_h-Q_c}{Q_h}=1-\frac{Q_c}{Q_h} \tag{1}$$

Since the absorbed heat Q_h is equal to the sum of the work done and the heat expelled, the heat engine itself returns back to its original state at the end of the process. By driving the engine in a cycle of such processes, a continuously operating engine can be obtained.

At this point, it might seem that by merely arranging for no heat to be expelled so that $Q_c=0$, we could develop a perfectly efficient engine that converts all its input heat into work. However, the second law of thermodynamics throws a spanner into the works, since it requires that entropy increases in any process that involves heat transfer. From the macroscopic definition of entropy, the amount of entropy absorbed from the hot reservoir is

$$S_h=-\frac{Q_h}{T_h} \tag{2}$$

(I've written it with a minus sign, since the entropy of the hot reservoir decreases.) The amount expelled into the cold reservoir is

$$S_c=\frac{Q_c}{T_c} \tag{3}$$

The overall change in entropy must be positive, so

$$S_h + S_c = \frac{Q_c}{T_c} - \frac{Q_h}{T_h} > 0 \tag{4}$$

In other words

$$\frac{Q_c}{Q_h} > \frac{T_c}{T_h} \tag{5}$$

Thus the only way Q_c can be zero (to get 100% efficiency) is if the cold reservoir is at absolute zero. In fact, the best we can do is to get an efficiency of

$$e \le 1 - \frac{T_c}{T_h} \tag{6}$$

As an example, suppose we look again at the monatomic ideal gas that is driven around a rectangular cycle on a PV diagram. To review, the system is as follows. The path is a rectangle starting at P_1 and V_1. On side A, the pressure is increased to P_2 while the volume is held constant. Then on side B, the volume is increased to V_2 while the pressure is constant at P_2. On side C, the pressure is decreased back to P_1 with the volume constant at V_2. Finally, on side D, the volume is decreased back to V_1 with the pressure constant at P_1. From the table in the earlier post, we see that heat is added to the gas along sides A and B, and removed from the gas along sides C and D. The net work done by the gas is given by

$$W = Q_A + Q_B + Q_C + Q_D \tag{7}$$

where $Q_C < 0$ and $Q_D < 0$ to indicate that heat is leaving the gas. We can calculate the efficiency of this cycle if it is used as a heat engine:

$$\begin{aligned} e &= 1 - \frac{Q_{expelled}}{Q_{absorbed}} && (8) \\ &= 1 - \frac{-Q_C - Q_D}{Q_A + Q_B} && (9) \end{aligned}$$

For the particular case $P_2 = 2P_1$ and $V_2 = 3V_1$ given in the problem, and reading the formulas for the heats off the table in the previous post, we get

$$\begin{aligned}Q_A &= \frac{5}{2}V_1(P_2-P_1)=2.5P_1V_1 &(10)\\ Q_B &= \frac{7}{2}P_2(V_2-V_1)=14P_1V_1 &(11)\\ -Q_C &= \frac{5}{2}V_2(P_2-P_1)=7.5P_1V_1 &(12)\\ -Q_D &= \frac{7}{2}P_1(V_2-V_1)=7P_1V_1 &(13)\end{aligned}$$

[The sign conventions can be a bit confusing, since Schroeder refers to all heat and work quantities as positive, whereas in the earlier chapters a distinction was made between heat absorbed by the gas and emitted by the gas.]

The efficiency is therefore

$$e=1-\frac{14.5}{16.5}=0.12 \tag{14}$$

This isn't a very efficient cycle for a heat engine.

In the course of the cycle, the gas varies between a low temperature of $T_c=P_1V_1/Nk$ and a high temperature of $T_h=P_2V_2/Nk=6T_c$. Thus the theoretical maximum efficiency is

$$e_{max}=1-\frac{1}{6}=0.83 \tag{15}$$

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