

## POWER PLANTS AS HEAT ENGINES

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 4.2 - 4.3.

Most power generating plants are versions of heat engines, in that some fuel (coal, gas or nuclear fission) is used to heat up a reservoir to a temperature  $T_h$  from which the generators extract an amount of heat  $Q_h$ , converting an amount  $W$  into electric power and expelling the remainder  $Q_c = Q_h - W$  as waste heat into a cold reservoir at temperature  $T_c$ .

The efficiency  $e$  of the heat engine is the fraction of  $Q_h$  that is converted into work, so

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \quad (1)$$

**Example 1.** Suppose a plant produces 1 GW ( $10^9$  W) of electricity by taking in steam at a temperature of  $T_h = 500^\circ \text{C} = 773 \text{ K}$  and expelling waste heat into the surrounding environment at a temperature of  $T_c = 20^\circ \text{C} = 293 \text{ K}$ . The maximum possible efficiency for an engine operating between these two temperatures is

$$e_{max} = 1 - \frac{T_c}{T_h} = 0.620 \quad (2)$$

If  $T_h$  could be raised to  $600^\circ \text{C} = 873 \text{ K}$ , the maximum efficiency increases to

$$e_{max} = 0.664 \quad (3)$$

To work out how much extra electricity this allows the plant to generate, we need to know the actual efficiency of the plant, which isn't given in the question, so presumably we are meant to assume that the plant is operating at maximum efficiency (which isn't realistic, since that would require a Carnot cycle, which would be far too slow to generate electricity). At any rate, we'll assume that this is the case, and as we're told that the amount of fuel consumed by the plant doesn't change, we can assume that the amount of heat absorbed is unchanged as well.

When the plant is operating at  $T_h = 500^\circ \text{C}$ , the amount of heat absorbed in a year is

$$Q_h = \frac{W}{e} = \frac{10^9 \times (3600 \times 24 \times 365)}{0.62} = 5.086 \times 10^{16} \text{ J} \quad (4)$$

With the same  $Q_h$  at the higher efficiency, the amount of electricity generated is

$$W = eQ_h = 0.664 \times 5.086 \times 10^{16} = 3.377 \times 10^{16} \text{ J} \quad (5)$$

The power generated is therefore

$$P = \frac{3.377 \times 10^{16}}{3600 \times 24 \times 365} = 1.071 \times 10^9 \text{ W} \quad (6)$$

The extra power is therefore 71000 kilowatts. To see how much more money can be made per year from this, we can take a typical electricity rate here in the UK of around 10p per kwh (around 15 cents in the US; Schroeder's price of 5 cents has been overtaken by inflation).

$$71 \times 10^3 \times 24 \times 365 \times (0.10) = 62 \times 10^6 \quad (7)$$

The extra income is around £62 million.

**Example 2.** Now we have a more realistic 1 GW power plant that operates at an efficiency of 40%. It absorbs heat at a rate of (I'll use the same symbols as above to represent the rates of heat and work (with units of watts) rather than the actual amounts of heat and work, to avoid introducing a bunch of new symbols):

$$Q_h = \frac{W}{e} = \frac{10^9}{0.4} = 2.5 \times 10^9 \text{ W} \quad (8)$$

The rate at which it expels heat into the cold reservoir is

$$Q_c = Q_h - W = 1.5 \times 10^9 \text{ W} \quad (9)$$

If this waste heat is expelled into a river with a flow rate of  $100 \text{ m}^3 \text{ s}^{-1}$ , we can work out how much the temperature of the river increases as a result. The specific heat capacity of water is  $c = 4181 \text{ J kg}^{-1} \text{ K}^{-1}$  and in one second,  $100 \text{ m}^3$  of water flows past the plant, absorbing  $1.5 \times 10^9 \text{ J}$  of energy. Each kilogram of water therefore absorbs

$$Q = \frac{1.5 \times 10^9}{100 \text{ m}^3 \times 1000 \text{ kg m}^{-3}} = 1.5 \times 10^4 \text{ J kg}^{-1} \quad (10)$$

The temperature of the river increases by

$$\Delta T = \frac{Q}{c} = 3.6 \text{ K} \quad (11)$$

An alternative method of cooling the plant is by evaporation of the water. Assuming that the river is initially at 293 K, we need a quantity of water so that when it is heated to  $100^\circ\text{C} = 373\text{ K}$  and then vaporized, this takes  $1.5 \times 10^9\text{ J s}^{-1}$ . For one kilogram of water, the latent heat of vaporization is  $2.258 \times 10^6\text{ J}$ , so if we vaporize a mass  $m$  of water starting at 293 K, this takes an amount of heat given by

$$Q = 4181m(373 - 293) + 2.258 \times 10^6 m \quad (12)$$

$$= 2.592 \times 10^6 m \text{ J kg}^{-1} \quad (13)$$

The first term is the heat required to heat the water to its boiling point and the second term is the heat required to vaporize it at the boiling point.

In one second, the mass of water required is therefore

$$m = \frac{1.5 \times 10^9}{2.592 \times 10^6} = 579 \text{ kg s}^{-1} \quad (14)$$

This represents a fraction of the total flow of the river of:

$$\frac{579}{100 \text{ m}^3 \times 1000 \text{ kg m}^{-3}} = 0.00579 \quad (15)$$

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