OCEAN WATER AS A HEAT ENGINE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.4.

A different kind of proposed power plant uses the thermal gradient of the ocean drive a heat engine. If the temperature of the water at the surface of the ocean in some (presumably fairly tropical; it never reaches that temperature here in Scotland) location is $T_h = 22^\circ$ C $= 295$ K and at the bottom is $T_c = 4^\circ$ C $= 277$ K, then the maximum efficiency of an engine operating between these two reservoirs is

$$e_{max} = 1 - \frac{T_c}{T_h} = 0.061 \tag{1}$$

This is a pretty dire efficiency. If we want to produce 1 GW of electricity, so the plant is comparable to land-based power plants, how much surface water would we need to process to extract the required heat Q_h? We have

$$e_{max} = \frac{W}{Q_h} \tag{2}$$

The amount of heat to be extracted per second is therefore

$$Q_h = \frac{10^9}{0.061} = 1.64 \times 10^{10} \text{ J s}^{-1} \tag{3}$$

The heat expelled to the cold reservoir is

$$Q_c = Q_h - W = 1.54 \times 10^{10} \text{ J s}^{-1} \tag{4}$$

To find the minimum volume of water that must be processed per second, we can try the following argument. [Disclaimer: I'm not 100% convinced this argument is correct, but it seems to make some sense. Comments welcome.]

The amount of water needed clearly depends on how much of a temperature drop the extraction of heat Q_h produces; the smaller the volume of water, the larger the temperature drop produced. The same is true (in reverse) when expelling heat Q_c into the cold layer; the smaller the volume of water, the larger the temperature increase.

Now, we can't *decrease* the temperature of the upper layer to a final temperature T_f that is lower than the final temperature of the cold layer, so the maximum temperature drop of the upper layer is to a value of T_f that is equal to the final temperature of the lower layer after Q_c is dumped into it. If we take the masses (I'll use mass instead of volume, since heat capacities are expressed in terms of mass) of water processed to be the same in both layers, then we have

$$T_h - T_f = \frac{Q_h}{mc} \quad (5)$$

$$T_f - T_c = \frac{Q_c}{mc} \quad (6)$$

where m is the mass of water processed in each layer and c is the specific heat capacity. We can add these two equations to get

$$T_h - T_c = \frac{Q_h + Q_m}{mc} \quad (7)$$

$$m = \frac{Q_h + Q_m}{c(T_h - T_c)} \quad (8)$$

Plugging in the numbers we get

$$m = \frac{(1.64 + 1.54) \times 10^{10}}{4181 \times 18} = 4.23 \times 10^5 \text{ kg} \quad (9)$$

This corresponds to a volume (since 1 m^3 of water has a mass of 1000 kg) of

$$V = 423 \text{ m}^3 \quad (10)$$

The *total* volume of water processed (hot + cold) is twice this, or

$$V_{total} = 846 \text{ m}^3 \quad (11)$$