REFRIGERATORS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 4.7 - 4.12.

A heat enginecan be run in reverse, so that it extracts an amount Q_c of heat from a cold reservoir by applying an amount W of work, and then expels the total $Q_h = Q_c + W$ as heat into a hot reservoir. This is the principle behind refrigeration as used, for example, in household refrigerators and air conditioners.

The efficiency of a refrigerator is measured as the ratio of the amount of heat extracted from the cold environment to the amount of work required to extract the heat. To avoid confusion with the efficiency of a heat engine, the efficiency of a refrigerator is called the *coefficient of performance* or COP for short. It is defined as

$$\text{COP} \equiv \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{Q_h/Q_c - 1} \tag{0.1}$$

From the second law of thermodynamics, the entropy extracted from the cold reservoir must be less than or equal to the entropy deposited in the hot reservoir, so

$$\frac{Q_c}{T_c} \leq \frac{Q_h}{T_h} \tag{0.2}$$

$$\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c} \tag{0.3}$$

Thus the maximum COP is obtained when we use the equality and we get

$$\text{COP} \leq \frac{1}{T_h/T_c - 1} = \frac{T_c}{T_h - T_c} \tag{0.4}$$

A larger COP is obtained if the temperature difference between the reservoirs is smaller, which is probably what we'd expect. If you want your kitchen fridge to cool its contents down only slightly below room temperature, it shouldn't take as much energy (work) as if you wanted to deep freeze things.

Example 1. An air conditioner works by using the room as the cold reservoir and the outside of the house as the hot reservoir. This is why you must put an air conditioner in a window so that it can extract heat from one side (the room) and expel it out the other (the outside). If you put it in the middle of the room then it is expelling the extracted heat back into the same environment, so it will have no effect.

Example 2. For exactly the same reason, you can't open a fridge door to cool down a room, since doing so just merges the hot and cold reservoirs into a single environment, and the fridge expels its heat into the same reservoir from which it extracts it.

Example 3. Estimating the maximum COP of a household air conditioner is a bit problematic for someone living in Scotland, since the summer temperature virtually never reaches a level where air conditioning is needed. A warm summer day here in Dundee is in the low 20s (centigrade; low 70s Fahrenheit). However, let's assume you live in a place where the outside temperature is 30° C $= 303$ K and you want your house to be at 22° C $=$ 295 K. Then the maximum COP is

$$\text{COP} = \frac{295}{303-295} = 36.875 \tag{0.5}$$

That is, you can extract heat at a rate about 37 times as great as the power needed to run the air conditioner.

Example 4. If heat seeps in to a kitchen fridge at a rate of 300 W, and the fridge has an average internal temperature of 0° C $= 273$ K in a room with temperature 22° C $= 295$ K, then the optimal COP is

$$\text{COP} = \frac{273}{295-273} = 12.4 \tag{0.6}$$

The electric power required to run the fridge is therefore $300/12.4 =$ 24 W. [In practice, fridges tend to use a lot more power than this. From Google, a typical 16 cubic foot fridge-freezer uses around 380 W. The difference is probably due to several factors, such as the fridge not having an optimal COP, extra energy being required to drive the compressor, and possibly a higher rate of heat entering the fridge.]

Example 5. For cryogenics, very low temperatures must be obtained. For refrigerator operating between a high-temperature reservoir of $T_h = 1$ K and a low temperature reservoir of $T_c = 0.01$ K, the maximum COP is

$$\text{COP} = \frac{0.01}{1-0.01} = 0.01 \tag{0.7}$$

Example 6. All the refrigerator examples we've seen so far rely on a cycle in which one phase has the working substance in contact with a hot reservoir where it expels heat and another phase where it is in contact with the cold reservoir, where it absorbs heat. Earlier, we looked at a cycle in which an ideal gas is driven around a cyclethat is rectangular on a PV diagram. To review, the system is as follows. The path is a rectangle starting at P_1 and V_1. On side A, the pressure is increased to P_2 while the volume is held constant. Then on side B, the volume is increased to V_2 while the pressure is constant at P_2. On side C, the pressure is decreased back to P_1 with the volume constant at V_2. Finally, on side D, the volume is decreased back to V_1 with the pressure constant at P_1. From the table in the earlier post, we see that heat is added to the gas along sides A and B, and removed from the gas along sides C and D.

Such a cycle can be used as a heat enginesince in each cycle, the gas absorbs an amount of heat $Q_A + Q_B$ and expels an amount $Q_C + Q_D$ where $Q_A + Q_B > Q_C + Q_D$ with the difference being the amount of work W the gas does on its environment.

It might seem that if we simply reversed the direction of the cycle, we could use such a system as a refrigerator, since the cycle would absorb $Q_C + Q_D$ from its environment and after inputting a net amount W of work (to compress the gas at the appropriate places), it would then expel $Q_A + Q_B > Q_C + Q_D$. To see why this won't work, we need to consider the temperature of the gas as it goes through these stages. From the ideal gas law $PV = NkT$, the lowest temperature in the cycle occurs at the lower left point on the rectangle where $T = P_1V_1/Nk$. In the reversed cycle, we proceed from this point along side D, where we increase the volume to V_2 while holding the pressure constant at P_1. This increases the temperature to $T = P_1V_2/Nk$, so the gas absorbs heat along this edge. Next, we proceed up side C, keeping V constant at V_2 and increasing the pressure to P_2, thus reaching the maximum temperature $T_{max} = P_2V_2/Nk$. Heat is also absorbed along this edge as we increase the pressure in a constant volume.

Now we proceed to the left along side B, decreasing the volume back to V_1 while keeping the pressure constant at P_2, thus decreasing the temperature to $T = P_2V_1/Nk$. During this stage, the gas expels heat. Finally we proceed down side A reducing the pressure back to P_1 while keeping the volume constant at V_1, thus reducing the temperature back to its starting value of $T = P_1V_1/Nk$. The gas expels more heat during this step.

However, notice that the temperature increases over its full range during the steps in which heat is absorbed, and then decreases over its full range during the steps where heat is expelled. In order for this to work, the gas would need to be colder than the cold environment over steps D and C while it is absorbing heat, and then hotter than the hot environment over steps B and A where it is expelling heat. But since it covers the same temperature range in the D/C steps as it does in the B/A steps, this is impossible.

In other words, a refrigerator really needs the two isothermal steps in its cycle, so it can absorb heat in one step and expel it in the other. This isn't as important for a heat engine since in that case, we don't really care that much about the temperature of the gas as it goes around a cycle; all we care about is that we can extract work from the cycle.

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