SUPER-EFFICIENT HEAT ENGINES AND REFRIGERATORS

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 4.16 - 4.17.

A heat engine has a maximum efficiency of

$$e_{max} = 1 - \frac{T_c}{T_h} \tag{1}$$

where T_h is the temperature of the hot reservoir from which heat Q_h is extracted and T_c is the temperature of the cold reservoir to which heat $Q_c = Q_h - W$ is expelled after work W is extracted. From the second law of thermodynamics, the heat and temperature ratios satisfy (at the optimum efficiency):

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h} \tag{2}$$

Thus the work done by an optimal heat engine is

$$W = Q_h - Q_c = Q_h \left(1 - \frac{T_c}{T_h}\right) \tag{3}$$

For an optimal refrigerator, the work required to extract Q_c from the cold reservoir and expel $Q_h = Q_c + W$ to the hot reservoir is exactly the same as 3. Therefore, if we use the work from a Carnot engine (which operates at the maximum efficiency) to power a Carnot refrigerator, the refrigerator will exactly cancel the effect of the heat engine, since it removes exactly the same amount Q_c of heat from the cold reservoir that the Carnot engine deposited there, and expels exactly the same amount Q_h of heat back into the hot reservoir that the Carnot engine extracted in the first place.

If, however, we could make an engine that was *more* efficient than a Carnot engine, that engine could produce the same amount of work by extracting a smaller amount of heat Q_{he} and expelling a smaller amount of waste heat Q_{ce}, provided that $Q_{he} - Q_{ce} = W$ as before. In this case, the heat engine still provides the amount of work required to power the refrigerator, but it produces *less* waste heat than the refrigerator extracts. The net effect of combining the two is to produce a system that requires no input of work, but still extracts a net amount $Q_c - Q_{ce}$ of heat from the cold reservoir. That is, we've produced a refrigerator that requires no input work.

Conversely, if we started with a refrigerator that had a coefficient of performance COP that is greater than the Carnot value of

$$\mathrm{COP}_{max} = \frac{T_c}{T_h - T_c} \tag{4}$$

this means that it requires an amount of work $W_r < W$ to extract the heat Q_c from the cold reservoir. Hooking this up to a Carnot engine that extracts Q_h from the hot reservoir, produces work W and expels heat Q_c to the cold reservoir means that the compound system produces a net excess of work $W - W_r$ while producing no waste heat, since the refrigerator extracts exactly the same amount of heat that the Carnot engine expels.

Both of these systems satisfy the first law of thermodynamics (conservation of energy) but violate the second law (entropy cannot decrease), which is why we can't make them in real life.