DIESEL ENGINES

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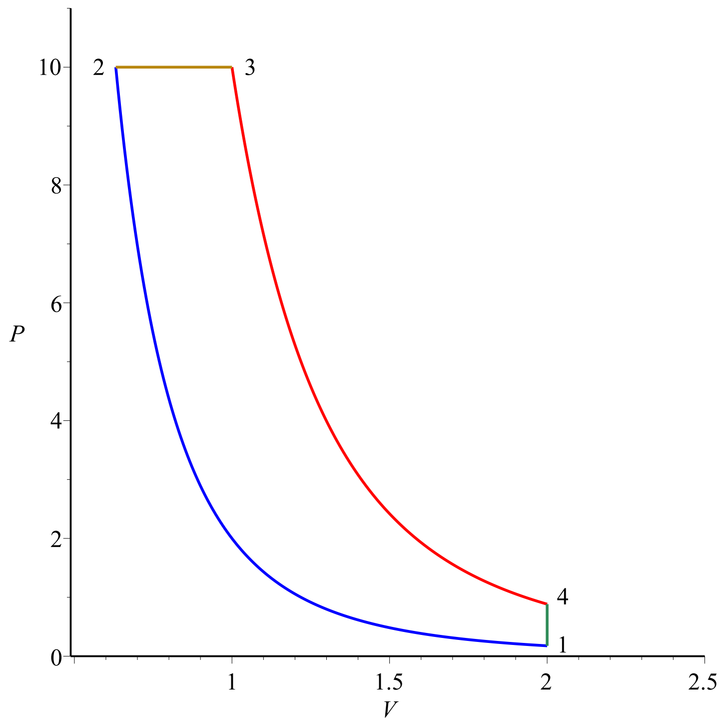
Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.20.

The efficiency of an internal combustion engine is

$$e=\frac{W}{Q_h}=1-\left(\frac{V_2}{V_1}\right)^{\gamma-1} \tag{1}$$

The ratio V_1/V_2 is known as the compression ratio (remember $V_1>V_2$ so this is always greater than 1), and is the ratio of the minimum to the maximum volume within a cylinder in the engine. It might seem that we could increase the efficiency as much as we like (well, up to the limit imposed by the second law of thermodynamics, anyway) simply by increasing the compression ratio. However, if the air/petrol mix is compressed beyond a certain point, it spontaneously ignites resulting in the ignition occurring before the volume has reached its minimum value of V_2, so there is a practical limit due to the chemistry.

A Diesel engine avoids this problem by compressing only the air and then injecting the fuel vapour when the desired pressure is reached, at which point the mixture spontaneously ignites. The Diesel cycle differs from the Otto cycle in that the ignition occurs at constant pressure rather than constant volume, so the diagram looks like this:



As in the Otto cycle, the red and blue curves are adiabats, so no heat is exchanged during these steps. The efficiency of the Diesel cycle is most easily worked out by calculating the heat input Q_h along the yellow step 2 to 3 and the heat expelled during the green step 4 to 1. The efficiency is then

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \tag{2}$$

[We could also calculate the net work by integrating along the red and blue curves like we did for the Otto cycle.]

The energy required for the constant pressure process 2 to 3 is the enthalpy difference between the two points. Remember that when a gas expands at constant pressure, both its volume and temperature increase, so the total energy required is the sum of the extra internal energy ΔU and the work $P\Delta V$ the gas does to expand. We have

$$
\begin{aligned}
Q_h &= \Delta U + P\Delta V && (3)\\
&= \frac{f}{2}Nk\left(T_3 - T_2\right) + P_2\left(V_3 - V_2\right) && (4)\\
&= \frac{1}{\gamma - 1}P_2\left(V_3 - V_2\right) + P_2\left(V_3 - V_2\right) && (5)\\
&= \frac{\gamma}{\gamma - 1}P_2\left(V_3 - V_2\right) && (6)\\
&= \frac{\gamma}{\gamma - 1}P_2V_2\left(\frac{V_3}{V_2} - 1\right) && (7)
\end{aligned}
$$

where f is the number of degrees of freedom per molecule and $\gamma = \left(f+2\right)/f$ is the adiabatic exponent. Subscripts refer to the numbered points on the graph.

The heat expelled occurs at constant volume, so we have

$$
\begin{aligned}
Q_c &= \frac{f}{2}Nk\left(T_4 - T_1\right) && (8)\\
&= \frac{V_1}{\gamma - 1}\left(P_4 - P_1\right) && (9)
\end{aligned}
$$

Since the red and blue curves are adiabats, we have

$$
\begin{aligned}
P_4V_1^{\gamma} &= P_2V_3^{\gamma} && (10)\\
P_1V_1^{\gamma} &= P_2V_2^{\gamma} && (11)
\end{aligned}
$$

Therefore

$$
Q_c = \frac{V_1P_2}{\gamma - 1}\left[\left(\frac{V_3}{V_1}\right)^{\gamma} - \left(\frac{V_2}{V_1}\right)^{\gamma}\right] \qquad (12)
$$

The efficiency is found by substituting into 2:

$$
\begin{aligned}
e &= 1-\frac{V_1\left[\left(\frac{V_3}{V_1}\right)^{\gamma}-\left(\frac{V_2}{V_1}\right)^{\gamma}\right]}{\gamma V_2\left(\frac{V_3}{V_2}-1\right)} && (13)\\
&= 1-\frac{V_1\left[\left(\frac{V_2}{V_1}\right)^{\gamma}\left(\frac{V_3}{V_2}\right)^{\gamma}-\left(\frac{V_2}{V_1}\right)^{\gamma}\right]}{\gamma V_2\left(\frac{V_3}{V_2}-1\right)} && (14)\\
&= 1-\left(\frac{V_2}{V_1}\right)^{\gamma-1}\frac{\left(\frac{V_3}{V_2}\right)^{\gamma}-1}{\gamma\left(\frac{V_3}{V_2}-1\right)} && (15)
\end{aligned}
$$

The ratio V_1/V_2 is the compression ratio as before, and the other ratio V_3/V_2 is the cutoff ratio. The Diesel efficiency is always lower than the Otto efficiency 1 which we can see as follows.

Since $V_3 \geq V_2$, the smallest possible value for V_3/V_2 is 1. Using l'Hôpital's rule we have

$$\lim_{x\to 1}\frac{x^{\gamma}-1}{\gamma(x-1)}=\lim_{x\to 1}\frac{\gamma x^{\gamma-1}}{\gamma}=1 \tag{16}$$

Since $\gamma = 1+\frac{2}{f}>1$, the function $\left(x^{\gamma}-1\right)/\left(x-1\right)$ is monotonically increasing for $x>1$ so

$$
\begin{aligned}
\left(\frac{V_2}{V_1}\right)^{\gamma-1}\frac{\left(\frac{V_3}{V_2}\right)^{\gamma}-1}{\gamma\left(\frac{V_3}{V_2}-1\right)} &> \left(\frac{V_2}{V_1}\right)^{\gamma-1} && (17)\\
e_{Diesel} &< e_{Otto} && (18)
\end{aligned}
$$

For $V_1/V_2 = 18$ and $V_3/V_2 = 2$ the efficiency is (using $\gamma = \frac{7}{5}$ for air)

$$e_{Diesel} = 0.632 \tag{19}$$

For the same compression ratio the Otto efficiency is

$$e_{Otto} = 0.685 \tag{20}$$

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