

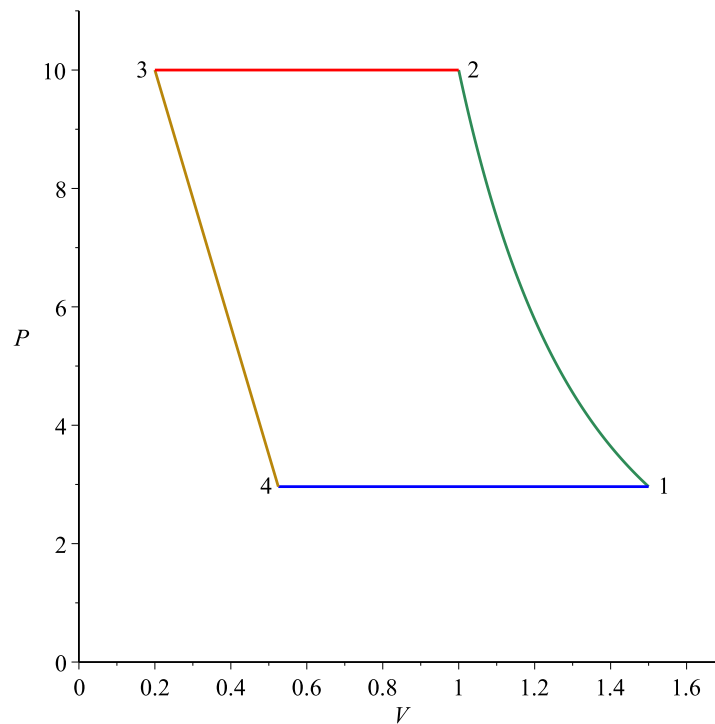
## THROTTLING: ENTHALPY VERSUS ENTROPY

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.31.

The edge  $3 \rightarrow 4$  of the cycle used in a refrigerator uses throttling to reduce the temperature of the working fluid:



Schroeder mentions that no heat is exchanged along this edge and as we saw in the earlier post, the enthalpy remains constant along the throttling edge, so that  $H_3 = H_4$ . Because no heat is exchanged, the process, by definition, is adiabatic. This might lead us to think that entropy is also constant during throttling, but this turns out not to be the case. Using the thermodynamic identity, we consider an infinitesimal throttling process:

$$(0.1) \quad TdS = dU + PdV$$

$$(0.2) \quad = dH - VdP$$

$$(0.3) \quad = -VdP$$

where we've used  $dH = 0$  since the enthalpy is constant. Thus the change in entropy is

$$(0.4) \quad dS = -\frac{V}{T}dP$$

which is positive, since the pressure decreases as we travel along the edge  $3 \rightarrow 4$ . Since no heat is exchanged,  $Q = 0$  so  $dS > Q/T$ , which means that new entropy is being created, so throttling is fundamentally an irreversible process.

Throttling isn't actually the most efficient design for a refrigeration cycle. Suppose we consider again the refrigerator in the earlier post, which operates between  $P_3 = 10$  bars and  $P_4 = 1$  bar. If we replace the throttling edge  $3 \rightarrow 4$  with an adiabatic, reversible expansion (so that entropy doesn't change), we can use the work it produces to run a turbine which can provide some of the power to run the compressor on the edge  $1 \rightarrow 2$ .

We can use Schroeder's Tables 4.3 and 4.4 to work out the enthalpies and thus the COP:

$$(0.5) \quad \text{COP} = \frac{H_1 - H_4}{H_2 - H_3 - (H_1 - H_4)}$$

$H_1, H_2$  and  $H_3$  are as before:

$$(0.6) \quad H_1 = 231 \text{ kJ}$$

$$(0.7) \quad H_2 = 279.08 \text{ kJ}$$

$$(0.8) \quad H_3 = 105 \text{ kJ}$$

To get  $H_4$  we use the fact that  $S_3 = S_4$  so from Table 4.3

$$(0.9) \quad S_4 = S_3 = 0.384 = 0.068x + 0.940(1 - x)$$

$$(0.10) \quad x = 0.638$$

$$(0.11) \quad H_4 = 16x + 231(1 - x)$$

$$(0.12) \quad = 93.9 \text{ kJ}$$

Thus  $H_4$  is a bit lower than  $H_3$  in this case, because the proportion of liquid at point 4 is higher. The new COP is, from 0.5:

$$(0.13) \quad \text{COP} = 3.71$$

This is almost as good as the ideal Carnot COP of 3.75 we worked out earlier.

I'm not sure why this method isn't used in real refrigerators. Perhaps a turbine causes too much noise?