THROTTLING: ENTHALPY VERSUS ENTROPY

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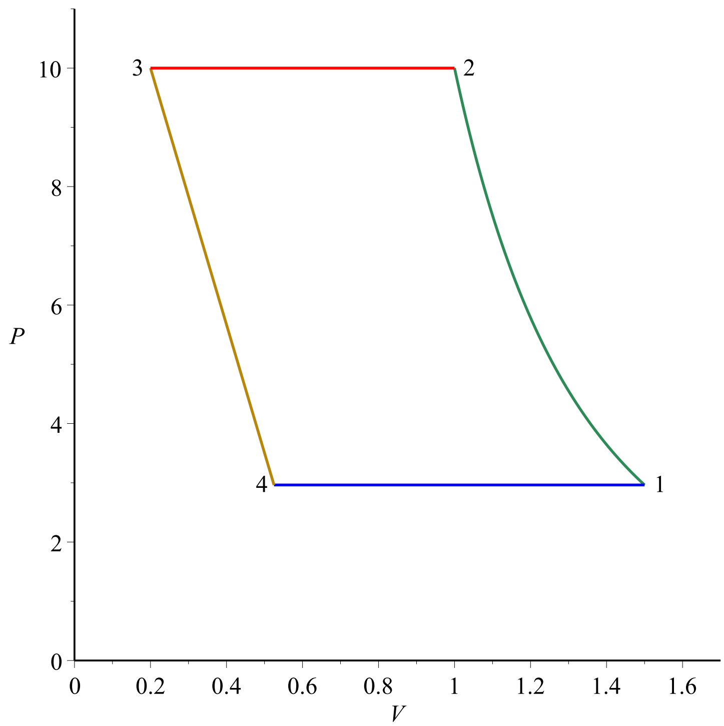
Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.31.

The edge $3 \to 4$ of the cycle used in a refrigerator uses throttling to reduce the temperature of the working fluid:



Schroeder mentions that no heat is exchanged along this edge and as we saw in the earlier post, the enthalpy remains constant along the throttling edge, so that $H_3 = H_4$. Because no heat is exchanged, the process, by definition, is adiabatic. This might lead us to think that entropy is also constant during throttling, but this turns out not to be the case. Using the thermodynamic identity, we consider an infinitesimal throttling process:

$$TdS = dU + PdV \tag{1}$$

$$= dH - VdP \tag{2}$$

$$= -VdP \tag{3}$$

where we've used $dH = 0$ since the enthalpy is constant. Thus the change in entropy is

$$dS = -\frac{V}{T}dP \tag{4}$$

which is positive, since the pressure decreases as we travel along the edge $3 \to 4$. Since no heat is exchanged, $Q = 0$ so $dS > Q/T$, which means that new entropy is being created, so throttling is fundamentally an irreversible process.

Throttling isn't actually the most efficient design for a refrigeration cycle. Suppose we consider again the refrigerator in theearlier post, which operates between $P_3 = 10$ bars and $P_4 = 1$ bar. If we replace the throttling edge $3 \to 4$ with an adiabatic, reversible expansion (so that entropy doesn't change), we can use the work it produces to run a turbine which can provide some of the power to run the compressor on the edge $1 \to 2$.

We can use Schroeder's Tables 4.3 and 4.4 to work out the enthalpies and thus the COP:

$$\mathrm{COP} = \frac{H_1 - H_4}{H_2 - H_3 - (H_1 - H_4)} \tag{5}$$

H_1, H_2 and H_3 are as before:

$$\begin{aligned} H_1 &= 231 \text{ kJ} && (6)\\ H_2 &= 279.08 \text{ kJ} && (7)\\ H_3 &= 105 \text{ kJ} && (8) \end{aligned}$$

To get H_4 we use the fact that $S_3 = S_4$ so from Table 4.3

$$\begin{aligned} S_4 = S_3 &= 0.384 = 0.068x + 0.940(1-x) && (9)\\ x &= 0.638 && (10)\\ H_4 &= 16x + 231(1-x) && (11)\\ &= 93.9 \text{ kJ} && (12) \end{aligned}$$

Thus H_4 is a bit lower than H_3 in this case, because the proportion of liquid at point 4 is higher. The new COP is, from 5:

$$\mathrm{COP} = 3.71 \tag{13}$$

This is almost as good as the ideal Carnot COP of 3.75 we worked out earlier.

I'm not sure why this method isn't used in real refrigerators. Perhaps a turbine causes too much noise?