

ENTHALPY IN THE HAMPSON-LINDE CYCLE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.34.

Gases such as nitrogen and oxygen can be liquefied using the Hampson-Linde cycle which uses the throttling process found in the real-world refrigeration cycle with the addition of a heat exchanger that allows the cooled low-pressure gas coming out of the throttling valve to absorb some heat from the high-pressure gas about to enter the valve.

Throttling on its own is a constant-enthalpy process, but if the heat exchanger in the Hampson-Linde cycle is ideal, so that the amount of heat Q absorbed by the low-pressure cooler gas is exactly equal to the amount of heat lost by the high-pressure warmer gas, the combination throttle + heat exchanger is still a constant enthalpy device. We can use the same argument as in the derivation of the throttling enthalpy. The difference in energy before and after the process is

$$U_f - U_i = (Q - P_f V_f) - (Q - P_i V_i) \quad (1)$$

As before, the PV terms come from the work done on or by the two pistons in the throttling valve. The first Q is the heat absorbed by the low-pressure gas after it leaves the throttle, and the second Q is the heat (numerically the same) lost by the high-pressure gas before it enters the throttle. These two heats cancel out, so the equation reduces to the one we had for the throttle on its own. Thus

$$U_f + P_f V_f = U_i + P_i V_i \quad (2)$$

and the enthalpies are equal.

We can use this fact to derive an equation for the fraction x of the gas that is liquefied in one pass through the cycle. The enthalpy H_i is the enthalpy H_{in} per mole of the incoming (high-pressure) gas. The final enthalpy $H_f = (1 - x)H_{out} + xH_{liq}$, where H_{out} is the enthalpy per mole of the low-pressure, cooler gas after it emerges from the throttle, and H_{liq} is the enthalpy per mole of any liquid that condenses. The total enthalpy after throttling is the sum, weighted according to the proportion of the gas that emerges in each state. Since the enthalpies are equal, we have

$$H_{in} = (1 - x)H_{out} + xH_{liq} \quad (3)$$

$$x = \frac{H_{out} - H_{in}}{H_{out} - H_{liq}} \quad (4)$$

Using data from Schroeder's Table 4.5 for nitrogen, if we start with a temperature of 300 K and assume that both the high-pressure and low-pressure gases have the same temperature (that is, the heat exchanger works perfectly) then, taking the liquid that condenses at its boiling point of 77 K:

$$x = \frac{8717 - 8174}{8717 + 3407} = 0.045 \quad (5)$$

For an initial temperature of 200 K, we get

$$x = \frac{5800 - 4442}{5800 + 3407} = 0.147 \quad (6)$$

I have to confess that I find this example very confusing. In the earlier calculation where we considered throttling on its own (without the heat exchanger), we found that throttling nitrogen from an initial temperature of 300 K resulted in the temperature dropping to 281.4 K, so as far as I can see, no liquid would be produced. I can't see how the introduction of a heat exchanger could improve things. Also, if the temperature of the gas emerging from the heat exchanger is the same that going into it, I can't see how the temperature would ever drop at all. In Schroeder's caption to his Figure 4.11, he says that it takes several cycles before liquid starts to be produced, which makes sense, since I would imagine that the heat absorbed by the cooler gas in the heat exchanger doesn't heat it back up to its original temperature, so when it is pumped back into the throttle, it starts at a lower temperature. Comments welcome.