MAGNETIC COOLING

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.35.

The common refrigeration cycle used in household refrigerators and the Hampson-Linde cycle used for liquefying gases such as nitrogen and oxygen don't work if we want to lower the temperature to under 1 K. There are several techniques that are used in this regime, one of which is *magnetic cooling*.

If we use our two-state paramagnet as a model, we can recall the formula for the magnetization of such a paramagnet:

$$M = N\mu \tanh\frac{\mu B}{kT} \tag{1}$$

Here, N is the number of dipoles, μ is the magnetic moment per dipole, B is the applied magnetic field and T is the temperature. If we apply a laboratory field B and obtain a value of $M/N\mu$ that is significantly higher than zero (zero magnetization means equal numbers of up and down dipoles, so if $M/N\mu > 0$ we have a majority of the dipoles pointing up). This ratio depends on the ratio B/T, so if we can maintain a constant magnetization as we decrease the applied magnetic field, the temperature will also drop.

The key to doing this appears to be to thermally insulate the sample as we lower the field. If no external heat can enter the system as B is decreased, there will be no change in the fraction of dipoles pointing up, so M remains the same. (At least I think this is the reason; Schroeder simply states that M remains constant as B is decreased without giving a reason.)

It might seem that since $\tanh 0 = 0$, we could obtain a temperature of absolute zero merely by switching off the external field completely, reducing B to zero. However, a magnetic dipole has its own magnetic field, given by

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[3\left(\mu\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}} - \mu\right] \tag{2}$$

where $\hat{\mathbf{r}}$ is a radial unit vector pointing from the dipole to the observation point, r is the distance to the observation point and μ is the dipole moment (written as a vector, since it does have a direction). Averaged over all directions, this field has a magnitude of

$$B = \frac{\mu_0 \mu}{4\pi r^3} \tag{3}$$

In a real substance, the dipoles will be aligned in various directions, so estimating the field felt by one dipole due to its neighbours is a bit tricky. Due to the $1/r^3$ falloff, however, we can probably neglect contributions from all but nearest neighbours. In a cubic lattice, there are 6 such neighbours, but to account for variations in directions of the dipoles, we'll take the number of neighbours to be, say, 3. Then the field at a dipole is

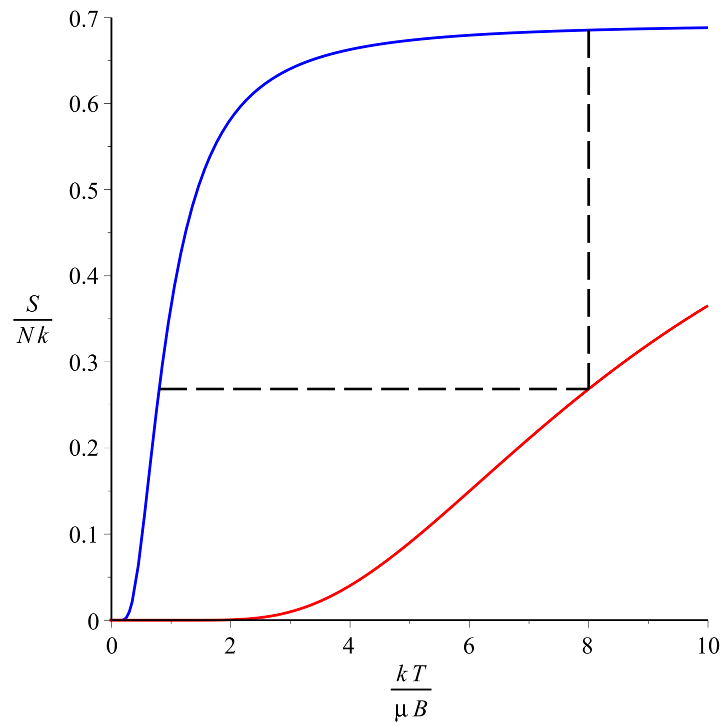
$$B \approx \frac{3\mu_0 \mu}{4\pi r^3} \tag{4}$$

For a substance where $\mu = 9 \times 10^{-24}$ J T^{-1} (roughly one Bohr magneton) and where the dipoles are 10^{-9} m apart, this gives an average field of (using $\mu_0 = 4\pi \times 10^{-7}$ in SI units):

$$B \approx \frac{3\left(10^{-7}\right)\left(9 \times 10^{-24}\right)}{\left(10^{-9}\right)^3} = 2.7 \times 10^{-3} \text{ T} \tag{5}$$

If we start with an applied field of 1 T and then switch this field off under conditions as described above, the field is reduced by a factor of $1/2.7 \times 10^{-3} = 370$, so the temperature would also be reduced by this factor. If we started at room temperature of 300 K, the final temperature would therefore be under 1 K after switching off the field.

In practice, the cooling technique follows the dotted path shown on the entropy-temperature diagram:



The blue and red curves are plots of the entropy as a function of temperature, which we found earlier as:

$$\frac{S}{Nk} = \ln\left(2\cosh\frac{1}{x}\right) - \frac{1}{x}\tanh\frac{1}{x} \tag{6}$$

where $x = kT/\mu B$.

The blue curve shows the entropy as a function of temperature for a low B field; the red curve for a high B field. The sample starts at a high temperature and low field, then is brought into contact with a thermal reservoir at the same temperature (to keep its temperature constant) while the field is increased, so we follow the vertical dashed line downwards. Then the sample is thermally insulated and the field is reduced back to the low value; the insulation ensures no heat flow, so S remains constant.

The cooling process aims for the steepest part of the blue curve, since here the heat capacity is highest. From $dU = T\,dS$ (at constant volume) we have

$$C_V = \frac{\partial U}{\partial T} = T\frac{\partial S}{\partial T} \tag{7}$$

A high heat capacity means that a relatively large amount of heat is required to change the temperature significantly, so at this point the system is least sensitive to heat leaking in from outside.

We can find the steepest point on the entropy-temperature plot by taking the second derivative of 6 with respect to T and setting to zero. This can't be solved analytically, but using Maple to get a numerical solution, we find that (for the blue curve)

$$x = 0.617 \tag{8}$$

At this point, using the dipole field 5, we have a temperature of

$$T = \frac{\mu Bx}{k} = \frac{\left(9\times10^{-24}\right)\left(2.7\times10^{-3}\right)(0.617)}{1.38\times10^{-23}} = 0.001\text{ K} \tag{9}$$

Thus using this technique, we should be able to achieve a temperature of around 1 mK. Going much lower would require lowering the horizontal dashed line in the plot down to near the horizontal axis where $\frac{\partial S}{\partial T}$ becomes very small since the graph flattens out here. This would cause heat leaks to be very significant.