LASER COOLING

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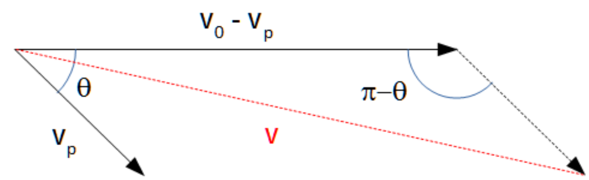
Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 4.36.

Another method of achieving very low temperatures is by *laser cooling*. The idea goes like this. Suppose we have a dilute gas of atoms, all of the same type. Each atom has a discrete set of energy levels that its electrons can occupy. If we have an atom at rest and tune our laser so that it emits photons of precisely the wavelength that can be absorbed by the atom, the atom will absorb the photon, making a transition to an excited state. Because a photon also carries momentum, the atom will recoil in the direction in which the original photon was travelling. Sometime later, the atom will spontaneously emit a photon of the same wavelength, although it will usually do this in some random direction (relative to the direction of the incident photon). Due to the recoil from the emission, the atom's velocity is now given a component opposite to the direction of emission, so that the vector sum of the recoil velocities of the atom from the two events will be in some random direction. (The vector sum of the final momenta of the atom and the emitted photon must, of course, add up to the momentum of the incident photon, by conservation of momentum.) Thus in this case, the final speed of the atom has increased slightly due to the absorption/emission process.

However, suppose we start with an atom in motion towards the laser, so that the incident photon makes a head-on collision with the atom. In this case, due to the Doppler effect, the atom sees the wavelength of the incident photon to be reduced slightly (blue-shifted), so its energy is no longer equal to the excitation energy of the atom, and the photon will not be absorbed. If we tune the laser to emit photons of a slightly lower energy, however, we can arrange things so that after taking the Doppler effect into account, the photon does have the correct energy to be absorbed by the atom. In this case, the atom's momentum is reduced by an amount equal to the momentum of the photon, so it slows down. Later, as before, the atom re-emits the photon in some random direction. To see what effect this has, suppose the photon is emitted at an angle $\pi-\theta$ to the direction of the incident photon, so that the atom recoils at an angle θ. A photon of wavelength λ has a momentum of $p_{\gamma}=E/c=h/\lambda$. When it is absorbed by an atom of mass m (we're assuming all speeds are non-relativistic), the atom's recoil momentum gives it a velocity component of

$$v_p = \frac{p_\gamma}{m} = \frac{h}{\lambda m} \tag{1}$$

The kick in momentum due to the recoil when the photon is re-emitted has the same magnitude, but occurs in direction θ, so the final velocity of the atom has a component $v_0 - v_p$ in the direction of the incident photon (where v_0 is the atom's original speed) and a component v_p in the direction θ. The situation is as shown:



The red line shows the resultant velocity v after emission of the photon. Using the cosine formula from trigonometry, we have

$$\begin{aligned} v^2 &= (v_0 - v_p)^2 + v_p^2 - 2(v_0 - v_p)\,v_p \cos(\pi - \theta) && (2) \\ &= (v_0 - v_p)^2 + v_p^2 + 2(v_0 - v_p)\,v_p \cos\theta && (3) \\ &= v_0^2 - 2v_p(v_0 - v_p)(1 - \cos\theta) && (4) \end{aligned}$$

From the last line, we see that $v < v_0$ provided $v_0 > v_p$, that is, the initial momentum of the atom is greater than the recoil it receives from the photon. In other words, the atom slows down after absorbing and re-emitting the photon. (Unless $\theta = 0$, in which case $v = v_0$ and the atom just moves along as before.)

The lowest speed we can attain as a result of laser cooling (for a fixed initial speed v_0) is thus when the photon's momentum is such that $v_p(v_0 - v_p)$ is a maximum which occurs when $v_p = v_0/2$. This gives a final v^2 of

$$v^2 = \frac{1}{2}v_0^2(1 + \cos\theta) \tag{5}$$

If we average this over $\theta = 0 \ldots \pi$ we get (since the average of $\cos\theta$ over that interval is zero):

$$\langle v^2 \rangle = \frac{v_0^2}{2} \tag{6}$$

Since the temperature is proportional to the kinetic energy, which is in turn proportional to v^2, the temperature of the gas is reduced by a factor of 2.

For a rubidium atom, we can work out the corresponding temperature from the equipartition theorem, since the kinetic energy of a single atom with only translational degrees of freedom is

$$U = \frac{3}{2}kT \tag{7}$$

For a photon of wavelength 780 nm $= 7.8 \times 10^{-7}$ m, the corresponding momentum is

$$p_\gamma = \frac{h}{\lambda} = 8.49 \times 10^{-28} \text{ kg m s}^{-1} \tag{8}$$

The mass of a rubidium atom is 85.5 amu, or

$$m = 85.5 \times 1.66 \times 10^{-27} = 1.42 \times 10^{-25} \text{ kg} \tag{9}$$

To get the maximum reduction in temperature, the initial momentum of the rubidium atom would need to be around $2p_\gamma$. The corresponding kinetic energy is

$$K = \frac{mv_0^2}{2} = \frac{4p_\gamma^2}{2m} \approx 10^{-29} \text{ J} \tag{10}$$

This corresponds to a final temperature of (using 6)

$$T = \frac{2}{3k}\frac{K}{2} \approx 2.5 \times 10^{-7} \text{ K} \tag{11}$$