MUSCLE AS A FUEL CELL

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problems 5.6 - 5.7.

As another example of a fuel cell, we'll look at the metabolism of glucose in an animal's muscle cells. The overall reaction is

$$C_6H_{12}O_6 + 6O_2 \to 6H_2O + 6CO_2 \tag{1}$$

The Gibbs free energy ΔG and enthalpy ΔH changes for this reaction can be obtained from the ΔG and ΔH values in Schroeder's book (all values for 1 mole at 298 K and 1 bar). As the reaction occurs at room temperature and pressure, we'll assume that the water product appears as a liquid rather than as a gas.

	ΔG (kJ)	ΔH (kJ)	S J K^{-1}
$C_6H_{12}O_6$	-910	-1273	212
O_2	0	0	205.14
H_2O	-237.13	-285.83	69.91
CO_2	-394.36	-393.51	213.74

The ΔG for the reaction is the sum of the values for the products minus the sum for the reactants:

$$\Delta G = 6(-237.13 - 394.36) - (6\times 0 - 910) = -2878.94 \text{ kJ mol}^{-1} \tag{2}$$

The value is per mole of glucose molecules.

The corresponding ΔH is found the same way:

$$\Delta H = 6(-285.83 - 393.51) - (6\times 0 - 1273) = -2803.04 \text{ kJ mol}^{-1} \tag{3}$$

The Gibbs energy represents the maximum energy that may be extracted as 'other' (that is, not due to volume changes) work, in this case chemical work. Thus we may extract up to 2878.94 kJ of electric work per mole of glucose metabolized.

As the reaction occurs at constant pressure, ΔH represents the *total* energy difference between the reactants and products. Since the enthalpy drop is less than the amount of work extracted, the difference must be absorbed as heat. The amount of heat is

$$Q = 2878.94 - 2803.04 = 75.9 \text{ kJ mol}^{-1} \tag{4}$$

The entropy increase resulting from absorbing this heat is

$$\Delta S = \frac{Q}{T} = \frac{75.9 \times 10^3}{298} = 254.7 \text{ J K}^{-1}\text{mol}^{-1} \tag{5}$$

If we work out the entropy change ΔS for this reaction using the values in the table above, we find

$$\Delta S = 6\,(69.91 + 213.74) - (6 \times 205.14 + 212) = 259.06 \text{ J K}^{-1}\text{mol}^{-1} \tag{6}$$

The values are roughly the same, so the absorption of heat can be explained by the entropy of the products being greater than that of the reactants.

This model assumes that the muscle is ideal, in the sense that all of the available ΔG is converted into chemical work in the muscle. If the muscle is less than ideal, then the amount of work performed is less than ΔG, so less heat is absorbed. However, the entropy difference between the reactants and products remains the same, so some of this entropy must be provided by means other than heat flow. This makes sense since a non-ideal muscle would use an irreversible process to perform its motion, resulting in an increase of entropy from other means.

The actual process by which glucose is metabolized is much more complicated than the simple reaction 1. In the process, 38 ATP (adensoine triphosphate) molecules are synthesized. When an ATP molecule splits into ADP (adenosine diphosphate) and a phosphate ion, it releases energy that is used in a variety of processes, including muscle contraction. The splitting of one ATP molecule provides energy for a molecule of myosin (an enzyme) to contract with a force of 4×10^{-12} N over a distance of 1.1×10^{-8} m. Thus one glucose molecule provides the energy for an amount of work

$$W = 38 \times 4 \times 10^{-12} \times 1.1 \times 10^{-8} = 1.672 \times 10^{-18} \text{ J} \tag{7}$$

The maximum amount of energy provided by one glucose molecule is obtained from 2 as

$$W_{max} = \frac{|\Delta G|}{6.02 \times 10^{23}} = 4.78 \times 10^{-18} \text{ J} \tag{8}$$

Thus the efficiency of muscle contraction is

$$e = \frac{W}{W_{max}} = 0.35 \tag{9}$$