ICE-WATER HEAT ENGINE

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 5.33.

A novel proposed heat engine uses the phase transition between water and ice to generate external work, based on the idea that water expands when it freezes. The engine consists of a vertical cylinder with cross sectional area A, filled with water and with a piston (assumed massless) at the top end. A mass m is placed on the piston. The cycle followed by the engine consists in first bringing the cylinder in contact with a cold reservoir at temperature T_c which is cold enough to freeze the water, causing the piston to rise and thus raising the weight through a distance h. The mass is then removed from the piston, and the system is brought in contact with a hot reservoir at temperature T_h which melts the ice, returning the engine to its starting point.

The catch with this engine is that adding the mass increases the pressure on the water, thus lowering its freezing point, as shown by the Clausius-Clapeyron relation:

$$\frac{dP}{dT}=\frac{L}{T\Delta V} \tag{1}$$

where L is the latent heat. When the ice melts, its volume changes by (water contracts when it melts)

$$\Delta V=-Ah \tag{2}$$

Adding the mass changes the pressure by

$$\Delta P=\frac{mg}{A} \tag{3}$$

Therefore, the freezing point is lowered by

$$\begin{aligned}\Delta T &= \frac{T\Delta V\Delta P}{L} &(4)\\ &= -mgh\frac{T}{L} &(5)\end{aligned}$$

The new freezing temperature is

$$T_c = T_0\left(1 - \frac{mgh}{L}\right) \tag{6}$$

where T_0 is the freezing point with no mass present (typically $T_0 =$ 273.15 K at standard conditions). We can see from this that if the mass m gets too large, T_c will become negative, so there is a limit to amount of weight such an engine can lift. (Of course, this assumes that the derivative 1 is constant over the range of pressure and temperature we're considering, which it probably isn't. However, the point is that there is a limit to the amount of pressure we can apply to ice before it melts.)

At the other end of the cycle, when we melt the ice, the mass has been removed so the melting point is unaffected and we have

$$T_h = T_0 \tag{7}$$

In one cycle, the engine absorbs heat $Q_h = L$ at the hot reservoir in order to melt the ice. An amount mgh of this heat is used to do the work to raise the mass through the distance h, so the amount of heat expelled to the cold reservoir is $Q_c = L - mgh$. The efficiency is

$$e = \frac{Q_h - Q_c}{Q_h} = \frac{mgh}{L} \tag{8}$$

From 6 and 7 we have

$$e = \frac{mgh}{L} = 1 - \frac{T_c}{T_0} = 1 - \frac{T_c}{T_h} \tag{9}$$

Thus the maximum efficiency of this engine is the same as that of a Carnot engine.